

Simplificación de fracciones algebraicas

EJERCICIO 1

1. $\frac{a^2}{ab} = \frac{a}{b}$

2. $\frac{2a}{8a^2b} = \frac{1}{4ab}$

3. $\frac{x^2y^2}{x^3y^3} = \frac{1}{xy}$

4. $\frac{ax^3}{4x^3y} = \frac{a}{4x^2y}$

5. $\frac{6m^2n^3}{3m} = 2mn^3$

6. $\frac{9x^2y^3}{24a^2x^3y^4} = \frac{3}{8a^2xy}$

7. $\frac{8m^4n^3x^2}{24mn^2x^2} = \frac{m^3n}{3}$

8. $\frac{12x^3y^4z^5}{32xy^2z} = \frac{3x^2y^2z^4}{8}$

9. $\frac{12a^2b^3}{60a^3b^5x^6} = \frac{1}{5ab^2x^6}$

10. $\frac{21mn^3x^6}{28m^4n^2x^2} = \frac{3nx^4}{4m^3}$

11. $\frac{42a^2c^3n}{26a^4c^5m} = \frac{21n}{13a^2c^2m}$

12. $\frac{7x^3y^4z^6}{34x^7y^8z^{10}} = \frac{1}{2x^4y^4z^4}$

13. $\frac{30x^6y^2}{45a^3x^4z^3} = \frac{2x^2y^2}{3a^3z^3}$

14. $\frac{a^5b^7}{3a^8b^9c} = \frac{1}{3a^3b^2c}$

15. $\frac{21a^8b^{10}c^{12}}{63a^4bc^2} = \frac{a^4b^9c^{10}}{3}$

16. $\frac{54x^9y^{11}z^{13}}{63x^{10}y^{12}z^{15}} = \frac{6}{7xyz^2}$

17. $\frac{15a^{12}b^{15}c^{20}}{75a^{11}b^{16}c^{22}} = \frac{a}{5bc^2}$

18. $\frac{75a^7m^5}{100a^3m^{12}n^3} = \frac{3a^4}{4m^7n^3}$

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EJERCICIO 2

$$1. \frac{3ab}{2a^2x+2a^3} = \frac{3ab}{2a^2(x+a)} = \frac{3b}{2a(x+a)}$$

$$2. \frac{xy}{3x^2y-3xy^2} = \frac{xy}{3xy(x-y)} = \frac{1}{3(x-y)}$$

$$3. \frac{2ax+4bx}{3ay+6by} = \frac{2x(a+2b)}{3y(a+2b)} = \frac{2x}{3y}$$

$$4. \frac{x^2-2x-3}{x-3} = \frac{(x-3)(x+1)}{x-3} = x+1$$

$$5. \frac{10a^2b^3c}{80(a^3-a^2b)} = \frac{a^2b^3c}{8a^2(a-b)} = \frac{b^3c}{8(a-b)}$$

$$6. \frac{x^2-4}{5ax+10a} = \frac{(x+2)(x-2)}{5a(x+2)} = \frac{x-2}{5a}$$

$$7. \frac{3x^2-4x-15}{x^2-5x+6} \\ = \frac{(3x)^2-4(3x)-45}{(x-3)(x-2)} \\ = \frac{[(3x-9)(3x+5)] \div 3}{(x-3)(x-2)} = \frac{(x-3)(3x+5)}{(x-3)(x-2)} = \frac{3x+5}{x-2}$$

$$8. \frac{15a^2bn-45a^2bm}{10a^2b^2n-30a^2b^2m} = \frac{15a^2b(n-3m)}{10a^2b^2(n-3m)} = \frac{3}{2b}$$

$$9. \frac{x^2-y^2}{x^2+2xy+y^2} = \frac{(x+y)(x-y)}{(x+y)^2} = \frac{x-y}{x+y}$$

$$10. \frac{3x^2y+15xy}{x^2-25} = \frac{3xy(x+5)}{(x+5)(x-5)} = \frac{3xy}{x-5}$$

$$11. \frac{a^2-4ab+4b^2}{a^3-8b^3} = \frac{(a-2b)^2}{(a-2b)(a^2+2ab+4b^2)} = \frac{a-2b}{a^2+2ab+4b^2}$$

$$12. \frac{x^3+4x^2-21x}{x^3-9x} = \frac{x(x^2+4x-21)}{x(x^2-9)} = \frac{(x+7)(x-3)}{(x+3)(x-3)} = \frac{x+7}{x+3}$$

$$13. \frac{6x^2+5x-6}{15x^2-7x-2} \\ = \frac{(6x)^2+5(6x)-36}{(15x)^2-7(15x)-30} \\ = \frac{[(6x+9)(6x-4)] \div 3 \cdot 2}{[(15x-10)(15x+3)] \div 5 \cdot 3} = \frac{(2x+3)(3x-2)}{(3x-2)(5x+1)} = \frac{2x+3}{5x+1}$$

$$14. \frac{a^3+1}{a^4-a^3+a-1} = \frac{a^3+1}{a^3(a-1)+(a-1)} = \frac{a^3+1}{(a^3+1)(a-1)} = \frac{1}{a-1}$$

15. $\frac{2ax + ay - 4bx - 2by}{ax - 4a - 2bx + 8b}$
 $= \frac{2x(a-2b) + y(a-2b)}{x(a-2b) - 4(a-2b)} = \frac{(2x+y)(a-2b)}{(x-4)(a-2b)} = \frac{2x+y}{x-4}$

16. $\frac{a^2 - ab - 6b^2}{a^3x - 6a^2bx + 9ab^2x}$
 $= \frac{(a-3b)(a+2b)}{ax(a^2 - 6b + 9b^2)} = \frac{(a-3b)(a+2b)}{ax(a-3b)^2} = \frac{a+2b}{ax(a-3b)}$

17. $\frac{m^2 + n^2}{m^4 - n^4} = \frac{m^2 + n^2}{(m^2 + n^2)(m^2 - n^2)} = \frac{1}{m^2 - n^2}$

18. $\frac{x^3 + y^3}{(x+y)^3} = \frac{(x+y)(x^2 - xy + y^2)}{(x+y)^3} = \frac{x^2 - xy + y^2}{(x+y)^2}$

19. $\frac{(m-n)^2}{m^2 - n^2} = \frac{(m-n)^2}{(m+n)(m-n)} = \frac{m-n}{m+n}$

20. $\frac{(a-x)^3}{a^3 - x^3} = \frac{(a-x)^3}{(a-x)(a^2 + ax + x^2)} = \frac{(a-x)^2}{a^2 + ax + x^2}$

21. $\frac{a^2 - a - 20}{a^2 - 7a + 10} = \frac{(a-5)(a+4)}{(a-5)(a-2)} = \frac{a+4}{a-2}$

22. $\frac{(1-a^2)^2}{a^2 + 2a + 1} = \frac{[(a+1)(1-a)]^2}{(a+1)^2} = (1-a)^2$

23. $\frac{a^4b^2 - a^2b^4}{a^4 - b^4} = \frac{a^2b^2(a^2 - b^2)}{(a^2 + b^2)(a^2 - b^2)} = \frac{a^2b^2}{a^2 + b^2}$

24. $\frac{x^2 - y^2}{x^3 - y^3} = \frac{(x+y)(x-y)}{(x-y)(x^2 + xy + y^2)} = \frac{x+y}{x^2 + xy + y^2}$

25. $\frac{24a^3b + 8a^2b^2}{36a^4 + 24a^3b + 4a^2b^2}$
 $= \frac{8a^2b(3a+b)}{4a^2(9a^2 + 6ab + b^2)} = \frac{2b(3a+b)}{(3a+b)^2} = \frac{2b}{3a+b}$

26. $\frac{n^3 - n}{n^2 - 5n - 6}$
 $= \frac{n(n^2 - 1)}{(n-6)(n+1)} = \frac{n(n+1)(n-1)}{(n-6)(n+1)} = \frac{n(n-1)}{n-6}$

27. $\frac{8n^3 + 1}{8n^3 - 4n^2 + 2n}$
 $= \frac{(2n+1)(4n^2 - 2n + 1)}{2n(4n^2 - 2n + 1)} = \frac{2n+1}{2n}$

28. $\frac{a^2 - (b-c)^2}{(a+b)^2 - c^2}$
 $= \frac{(a+b-c)(a-b+c)}{(a+b-c)(a+b+c)} = \frac{a-b+c}{a+b+c}$

29. $\frac{(a+b)^2 - (c-d)^2}{(a+c)^2 - (b-d)^2}$
 $= \frac{(a+b+c-d)(a+b-c+d)}{(a+b+c-d)(a-b+c+d)}$
 $= \frac{a+b-c+d}{a-b+c+d}$

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$$30. \frac{3x^3+9x^2}{x^2+6x+9} = \frac{3x^2(x+3)}{(x+3)^2} = \frac{3x^2}{x+3}$$

$$31. \frac{10a^2(a^3+b^3)}{6a^4-6a^3b+6a^2b^2} \\ = \frac{10a^2(a+b)(a^2-ab+b^2)}{6a^2(a^2-ab+b^2)} = \frac{5(a+b)}{3}$$

$$32. \frac{a(4a^2-8ab)}{x(3a^2-6ab)} = \frac{4a^2(a-2b)}{3ax(a-2b)} = \frac{4a}{3x}$$

$$33. \frac{x^3-6x^2}{x^2-12x+36} = \frac{x^2(x-6)}{(x-6)^2} = \frac{x^2}{x-6}$$

$$34. \frac{(x-4y)^2}{x^5-64x^2y^3} \\ = \frac{(x-4y)^2}{x^2(x^3-64y^3)} \\ = \frac{(x-4y)^2}{x^2(x-4y)(x^2+4xy+16y^2)} \\ = \frac{x-4y}{x^2(x^2+4xy+16y^2)}$$

$$35. \frac{x^3-3xy^2}{x^4-6x^2y^2+9y^4} = \frac{x(x^2-3y^2)}{(x^2-3y^2)^2} = \frac{x}{x^2-3y^2}$$

$$36. \frac{m^3n+3m^2n+9mn}{m^3-27} \\ = \frac{mn(m^2+3m+9)}{(m-3)(m^2+3m+9)} = \frac{mn}{m-3}$$

$$37. \frac{x^4-8x^2+15}{x^4-9} = \frac{(x^2-5)(x^2-3)}{(x^2+3)(x^2-3)} = \frac{x^2-5}{x^2+3}$$

$$38. \frac{a^4+6a^2-7}{a^4+8a^2-9} = \frac{(a^2+7)(a^2-1)}{(a^2+9)(a^2-1)} = \frac{a^2+7}{a^2+9}$$

$$39. \frac{3x^2+19x+20}{6x^2+17x+12} \\ = \frac{(3x)^2+19(3x)+60}{(6x)^2+17(6x)+72} \\ = \frac{[(3x+15)(3x+4)] \div 3}{[(6x+8)(6x+9)] \div 2 \cdot 3} \\ = \frac{(x+5)(3x+4)}{(3x+4)(2x+3)} = \frac{x+5}{2x+3}$$

$$\begin{aligned}
40. \quad & \frac{4a^4 - 15a^2 - 4}{a^2 - 8a - 20} \\
&= \frac{(4a^2)^2 - 15(4a^2) - 16}{(a-10)(a+2)} \\
&= \frac{[(4a^2 - 16)(4a^2 + 1)] \div 4}{(a-10)(a+2)} \\
&= \frac{(a^2 - 4)(4a^2 + 1)}{(a-10)(a+2)} \\
&= \frac{(a+2)(a-2)(4a^2 + 1)}{(a-10)(a+2)} \\
&= \frac{(a-2)(4a^2 + 1)}{a-10}
\end{aligned}$$

$$\begin{aligned}
41. \quad & \frac{125a + a^4}{2a^3 + 20a^2 + 50a} \\
&= \frac{a(125 + a^3)}{2a(a^2 + 10a + 25)} \\
&= \frac{(5+a)(25 - 5a + a^2)}{2(a+5)^2} \\
&= \frac{a^2 - 5a + 25}{2(a+5)}
\end{aligned}$$

$$\begin{aligned}
42. \quad & \frac{a^2n^2 - 36a^2}{an^2 + an - 30a} \\
&= \frac{a^2(n^2 - 36)}{a(n^2 + n - 30)} \\
&= \frac{a(n+6)(n-6)}{(n+6)(n-5)} \\
&= \frac{a(n-6)}{n-5}
\end{aligned}$$

$$\begin{aligned}
43. \quad & \frac{3m^2 + 5mn - 8n^2}{m^3 - n^3} \\
&= \frac{(3m)^2 + 5n(3m) - 24n^2}{(m-n)(m^2 + mn + n^2)} \\
&= \frac{[(3m+8n)(3m-3n)] \div 3}{(m-n)(m^2 + mn + n^2)} \\
&= \frac{(3m+8n)(m-n)}{(m-n)(m^2 + mn + n^2)} \\
&= \frac{3m+8n}{m^2 + mn + n^2}
\end{aligned}$$

$$\begin{aligned}
44. \quad & \frac{15a^3b - 18a^2b}{20a^2b^2 - 24ab^2} \\
&= \frac{3a^2b(5a-6)}{4ab^2(5a-6)} = \frac{3a}{4b}
\end{aligned}$$

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45. $\frac{9x^2 - 24x + 16}{9x^4 - 16x^2}$
 $= \frac{(3x-4)^2}{x^2(9x^2-16)}$
 $= \frac{(3x-4)^2}{x^2(3x+4)(3x-4)} = \frac{3x-4}{x^2(3x+4)}$

46. $\frac{16a^2x - 25x}{12a^3 - 7a^2 - 10a}$
 $= \frac{x(16a^2 - 25)}{a(12a^2 - 7a - 10)}$
 $= \frac{x(4a+5)(4a-5)}{a[(12a)^2 - 7(12a) - 120]}$
 $= \frac{x(4a+5)(4a-5)}{a[(12a-15)(12a+8)] \div 3 \cdot 4}$
 $= \frac{x(4a+5)(4a-5)}{a(4a-5)(3a+2)} = \frac{x(4a+5)}{a(3a+2)}$

47. $\frac{8x^4 - xy^3}{4x^4 - 4x^3y + x^2y^2}$
 $= \frac{x(8x^3 - y^3)}{x^2(4x^2 - 4xy + y^2)}$
 $= \frac{(2x-y)(4x^2 + 2xy + y^2)}{x(2x-y)^2}$
 $= \frac{4x^2 + 2xy + y^2}{x(2x-y)}$

48. $\frac{3an - 4a - 6bn + 8b}{6n^2 - 5n - 4}$
 $= \frac{3n(a-2b) - 4(a-2b)}{(6n)^2 - 5(6n) - 24}$
 $= \frac{(3n-4)(a-2b)}{[(6n-8)(6n+3)] \div 2 \cdot 3}$
 $= \frac{(3n-4)(a-2b)}{(3n-4)(2n+1)} = \frac{a-2b}{2n+1}$

49. $\frac{x^4 - 49x^2}{x^3 + 2x^2 - 63x}$
 $= \frac{(x^2+7x)(x^2-7x)}{x(x^2+2x-63)}$
 $= \frac{x^2(x+7)(x-7)}{x(x+9)(x-7)} = \frac{x(x+7)}{x+9}$

50. $\frac{x^4 + x - x^3y - y}{x^3 - x - x^2y + y}$
 $= \frac{x^3(x-y) + (x-y)}{x^2(x-y) - (x-y)}$
 $= \frac{(x^3+1)(x-y)}{(x^2-1)(x-y)}$
 $= \frac{(x+1)(x^2-x+1)}{(x+1)(x-1)}$
 $= \frac{x^2-x+1}{x-1}$

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$$\begin{aligned}
 51. \quad & \frac{2x^3 + 6x^2 - x - 3}{x^3 + 3x^2 + x + 3} \\
 &= \frac{2x^2(x+3) - (x+3)}{x^2(x+3) + (x+3)} \\
 &= \frac{(2x^2 - 1)(x+3)}{(x^2 + 1)(x+3)} = \frac{2x^2 - 1}{x^2 + 1}
 \end{aligned}$$

$$\begin{aligned}
 52. \quad & \frac{a^3m - 4am + a^3n - 4an}{a^4 - 4a^3 - 12a^2} \\
 &= \frac{am(a^2 - 4) + an(a^2 - 4)}{a^2(a^2 - 4a - 12)} \\
 &= \frac{a(m+n)(a^2 - 4)}{a^2(a-6)(a+2)} \\
 &= \frac{(m+n)(a+2)(a-2)}{a(a-6)(a+2)} \\
 &= \frac{(m+n)(a-2)}{a(a-6)}
 \end{aligned}$$

$$\begin{aligned}
 53. \quad & \frac{4a^2 - (x-3)^2}{(2a+x)^2 - 9} \\
 &= \frac{(2a+x-3)(2a-x+3)}{(2a+x+3)(2a+x-3)} \\
 &= \frac{2a-x+3}{2a+x+3}
 \end{aligned}$$

$$\begin{aligned}
 54. \quad & \frac{m - am + n - an}{1 - 3a + 3a^2 - a^3} \\
 &= \frac{m(1-a) + n(1-a)}{(1-a^3) - 3a(1-a)} \\
 &= \frac{(m+n)(1-a)}{(1-a)(1+a+a^2) - 3a(1-a)} \\
 &= \frac{(m+n)(1-a)}{(1-a)(1-2a+a^2)} \\
 &= \frac{m+n}{a^2 - 2a + 1} = \frac{m+n}{(a-1)^2}
 \end{aligned}$$

$$\begin{aligned}
 55. \quad & \frac{6x^2 + 3}{42x^5 - 9x^3 - 15x} \\
 &= \frac{3(2x^2 + 1)}{3x(14x^4 - 3x^2 - 5)} \\
 &= \frac{2x^2 + 1}{x[(14x^2)^2 - 3(14x^2) - 70]} \\
 &= \frac{2x^2 + 1}{x[(14x^2 - 10)(14x^2 + 7)] \div 2 \cdot 7} \\
 &= \frac{2x^2 + 1}{x(7x^2 - 5)(2x^2 + 1)} = \frac{1}{x(7x^2 - 5)}
 \end{aligned}$$

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$$\begin{aligned}
 56. \quad & \frac{a^2 - a^3 - 1 + a}{a^2 + 1 - a^3 - a} \\
 &= \frac{a^2(1-a) - (1-a)}{a^2(1-a) + (1-a)} \\
 &= \frac{(a^2-1)(1-a)}{(a^2+1)(1-a)} = \frac{a^2-1}{a^2+1}
 \end{aligned}$$

$$\begin{aligned}
 57. \quad & \frac{8x^3 + 12x^2y + 6xy^2 + y^3}{6x^2 + xy - y^2} \\
 &= \frac{6xy(2x+y) + (8x^3 + y^3)}{(6x)^2 + y(6x) - 6y^2} \\
 &= \frac{6xy(2x+y) + (2x+y)(4x^2 - 2xy + y^2)}{[(6x+3y)(6x-2y)] \div 3 \cdot 2} \\
 &= \frac{(2x+y)(4x^2 + 4xy + y^2)}{(2x+y)(3x-y)} = \frac{(2x+y)^2}{3x-y}
 \end{aligned}$$

$$\begin{aligned}
 58. \quad & \frac{8n^3 - 125}{25 - 20n + 4n^2} \\
 &= \frac{(2n-5)(4n^2 + 10n + 25)}{(2n-5)^2} \\
 &= \frac{4n^2 + 10n + 25}{2n-5}
 \end{aligned}$$

$$\begin{aligned}
 59. \quad & \frac{6 - x - x^2}{15 + 2x - x^2} \\
 &= \frac{-(x^2 + x - 6)}{-(x^2 - 2x - 15)} \\
 &= \frac{(x+3)(x-2)}{(x-5)(x+3)} = \frac{x-2}{x-5}
 \end{aligned}$$

$$\begin{aligned}
 60. \quad & \frac{3 + 2x - 8x^2}{4 + 5x - 6x^2} \\
 &= \frac{-(8x^2 - 2x - 3)}{-(6x^2 - 5x - 4)} \\
 &= \frac{(8x)^2 - 2(8x) - 24}{(6x)^2 - 5(6x) - 24} \\
 &= \frac{[(8x-6)(8x+4)] \div 2 \cdot 4}{[(6x-8)(6x+3)] \div 2 \cdot 3} \\
 &= \frac{(4x-3)(2x+1)}{(3x-4)(2x+1)} = \frac{4x-3}{3x-4}
 \end{aligned}$$

$$\begin{aligned}
 61. \quad & \frac{m^2n^2 + 3mn - 10}{4 - 4mn + m^2n^2} \\
 &= \frac{(mn+5)(mn-2)}{(mn-2)^2} = \frac{mn+5}{mn-2}
 \end{aligned}$$

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$$\begin{aligned}
 62. \quad & \frac{x^3 + x^2y - 4b^2x - 4b^2y}{4b^2 - 4bx + x^2} \\
 &= \frac{x^2(x+y) - 4b^2(x+y)}{(2b-x)^2} \\
 &= \frac{(x^2 - 4b^2)(x+y)}{(x-2b)^2} \\
 &= \frac{(x+2b)(x-2b)(x+y)}{(x-2b)^2} \\
 &= \frac{(x+2b)(x+y)}{x-2b}
 \end{aligned}$$

$$\begin{aligned}
 63. \quad & \frac{x^6 + x^3 - 2}{x^4 - x^3y - x + y} \\
 &= \frac{(x^3+2)(x^3-1)}{x^3(x-y) - (x-y)} \\
 &= \frac{(x^3+2)(x^3-1)}{(x^3-1)(x-y)} = \frac{x^3+2}{x-y}
 \end{aligned}$$

$$\begin{aligned}
 64. \quad & \frac{(x^2 - x - 2)(x^2 - 9)}{(x^2 - 2x - 3)(x^2 + x - 6)} \\
 &= \frac{(x-2)(x+1)(x+3)(x-3)}{(x-3)(x+1)(x+3)(x-2)} = 1
 \end{aligned}$$

$$\begin{aligned}
 65. \quad & \frac{(a^2 - 4a + 4)(4a^2 - 4a + 1)}{(a^2 + a - 6)(2a^2 - 5a + 2)} \\
 &= \frac{(a-2)^2(2a-1)^2}{(a+3)(a-2)[(2a)^2 - 5(2a) + 4]} \\
 &= \frac{(a-2)(2a-1)^2}{(a+3)[(2a-4)(2a-1)] \div 2} \\
 &= \frac{(a-2)(2a-1)^2}{(a+3)(a-2)(2a-1)} = \frac{2a-1}{a+3}
 \end{aligned}$$

$$\begin{aligned}
 66. \quad & \frac{(x^3 - 3x)(x^3 - 1)}{(x^4 + x^3 + x^2)(x^2 - 1)} \\
 &= \frac{x(x^2 - 3)(x-1)(x^2 + x + 1)}{x^2(x^2 + x + 1)(x+1)(x-1)} \\
 &= \frac{x^2 - 3}{x(x+1)}
 \end{aligned}$$

$$\begin{aligned}
 67. \quad & \frac{(4n^2 + 4n - 3)(n^2 + 7n - 30)}{(2n^2 - 7n + 3)(4n^2 + 12n + 9)} \\
 &= \frac{[(4n)^2 + 4(4n) - 12](n+10)(n-3)}{[(2n)^2 - 7(2n) + 6](2n+3)^2} \\
 &= \frac{(n+10)(n-3)[(4n+6)(4n-2)] \div 2 \cdot 2}{(2n+3)^2 [(2n-6)(2n-1)] \div 2} \\
 &= \frac{(n+10)(n-3)(2n+3)(2n-1)}{(2n+3)^2(n-3)(2n-1)} = \frac{n+10}{2n+3}
 \end{aligned}$$

$$\begin{aligned}
 68. \quad & \frac{(x^6 - y^6)(x+y)}{(x^3 - y^3)(x^3 + x^2y + xy^2 + y^3)} \\
 &= \frac{(x^3 + y^3)(x^3 - y^3)(x+y)}{(x^3 - y^3)[x^2(x+y) + y^2(x+y)]} \\
 &= \frac{(x^3 + y^3)(x+y)}{(x^2 + y^2)(x+y)} = \frac{x^3 + y^3}{x^2 + y^2}
 \end{aligned}$$

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EJERCICIO 3

$$1. \frac{4-4x}{6x-6} = \frac{4(1-x)}{6(x-1)} = -\frac{4(x-1)}{6(x-1)} = -\frac{2}{3}$$

$$2. \frac{a^2-b^2}{b^2-a^2} = -\frac{(a^2-b^2)}{(a^2-b^2)} = -1$$

$$3. \frac{m^2-n^2}{(n-m)^2} = \frac{(m+n)(m-n)}{(n-m)(n-m)} = \frac{(m+n)(m-n)}{(m-n)(m-n)} = \frac{m+n}{m-n}$$

$$4. \frac{x^2-x-12}{16-x^2} = \frac{(x-4)(x+3)}{(4-x)(4+x)} = -\frac{(x-4)(x+3)}{(x-4)(4+x)} = -\frac{x+3}{4+x}$$

$$5. \frac{3y-6x}{2mx-my-2nx+ny} \\ = \frac{3(y-2x)}{2x(m-n)-y(m-n)} \\ = \frac{3(y-2x)}{(2x-y)(m-n)} = -\frac{3(2x-y)}{(2x-y)(m-n)} = -\frac{3}{m-n}$$

$$6. \frac{2x^2-9x-5}{10+3x-x^2} \\ = -\frac{[(2x)^2-9(2x)-10]}{x^2-3x-10} \\ = -\frac{(2x-10)(2x+1)}{(x-5)(x+2)} = -\frac{(x-5)(2x+1)}{(x-5)(x+2)} = -\frac{2x+1}{x+2}$$

$$7. \frac{8-a^3}{a^2+2a-8} \\ = \frac{(2-a)(4+2a+a^2)}{(a+4)(a-2)} = -\frac{(a-2)(4+2a+a^2)}{(a+4)(a-2)} = -\frac{a^2+2a+4}{a+4}$$

$$8. \frac{a^2+a-2}{n-an-m+am} \\ = \frac{(a+2)(a-1)}{n(1-a)-m(1-a)} = \frac{(a+2)(a-1)}{(n-m)(1-a)} = \frac{(a+2)(a-1)}{(m-n)(a-1)} = \frac{a+2}{m-n}$$

$$9. \frac{4x^2-4xy+y^2}{5y-10x} = \frac{(2x-y)^2}{5(y-2x)} = -\frac{(2x-y)^2}{5(2x-y)} = -\frac{2x-y}{5}$$

$$10. \frac{3mx-nx-3my+ny}{ny^2-nx^2-3my^2+3mx^2} \\ = \frac{3m(x-y)-n(x-y)}{-3m(y^2-x^2)+n(y^2-x^2)} \\ = \frac{(3m-n)(x-y)}{(n-3m)(y^2-x^2)} = \frac{(3m-n)(x-y)}{(3m-n)(x^2-y^2)} = \frac{x-y}{(x+y)(x-y)} = \frac{1}{x+y}$$

$$11. \frac{9-6x+x^2}{x^2-7x+12} = \frac{(x-3)^2}{(x-4)(x-3)} = \frac{x-3}{x-4}$$

$$12. \frac{a^2-b^2}{b^3-a^3} \\ = \frac{(a+b)(a-b)}{(b-a)(b^2+ab+a^2)} \\ = -\frac{(a+b)(a-b)}{(a-b)(b^2+ab+a^2)} = -\frac{a+b}{b^2+ab+a^2}$$

$$13. \frac{3ax-3bx-6a+6b}{2b-2a-bx+ax} \\ = \frac{3x(a-b)-6(a-b)}{2(b-a)-x(b-a)} \\ = \frac{3(x-2)(a-b)}{(2-x)(b-a)} = \frac{3(x-2)(a-b)}{(x-2)(a-b)} = 3$$

$$14. \frac{a^2-x^2}{x^2-ax-3x+3a} \\ = \frac{(a+x)(a-x)}{x(x-a)-3(x-a)} \\ = \frac{(a+x)(a-x)}{(x-3)(x-a)} = \frac{(a+x)(a-x)}{(3-x)(a-x)} = \frac{a+x}{3-x}$$

$$15. \frac{3bx-6x}{8-b^3} \\ = \frac{3x(b-2)}{(2-b)(4+2b+b^2)} \\ = -\frac{3x(b-2)}{(b-2)(4+2b+b^2)} = -\frac{3x}{4+2b+b^2}$$

$$16. \frac{(1-a)^3}{a-1} = -\frac{(1-a)^3}{(1-a)} = -(1-a)^2$$

$$17. \frac{2x^3-2x^2y-2xy^2}{3y^3+3xy^2-3x^2y} \\ = \frac{2x(x^2-xy-y^2)}{3y(y^2+xy-x^2)} = -\frac{2x(x^2-xy-y^2)}{3y(x^2-xy-y^2)} = -\frac{2x}{3y}$$

$$18. \frac{(a-b)^3}{(b-a)^2} = \frac{(a-b)^3}{(a-b)^2} = a-b$$

$$19. \frac{2x^2-22x+60}{75-3x^2} \\ = \frac{[(2x)^2-22(2x)+120]}{3(25-x^2)} \\ = \frac{(2x-12)(2x-10)}{3(5-x)(5+x)} \\ = \frac{(2x-12)(x-5)}{3(5-x)(5+x)} = \frac{2(6-x)(5-x)}{3(5-x)(5+x)} = \frac{2(6-x)}{3(5+x)}$$

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$$20. \frac{6an^2 - 3b^2n^2}{b^4 - 4ab^2 + 4a^2} = \frac{3n^2(2a - b^2)}{(b^2 - 2a)^2} = \frac{3n^2(2a - b^2)}{(2a - b^2)(2a - b^2)} = \frac{3n^2}{2a - b^2}$$

$$21. \frac{(x - y)^2 - z^2}{(y + z) - x^2} = \frac{(x - y - z)(x - y + z)}{(y + z - x)(y + z + x)} = \frac{(z + y - x)(y - x - z)}{(z + y - x)(y + z + x)} = \frac{y - x - z}{x + y + z}$$

$$22. \frac{3a^2 - 3ab}{bd - ad - bc + ac} = \frac{3a(a - b)}{d(b - a) - c(b - a)} = \frac{3a(a - b)}{(d - c)(b - a)} = \frac{3a(a - b)}{(c - d)(a - b)} = \frac{3a}{c - d}$$

$$23. \frac{(x - 5)^3}{125 - x^3} = \frac{(x - 5)^3}{(5 - x)(25 + 5x + x^2)} = -\frac{(x - 5)^3}{(x - 5)(x^2 + 5x + 25)} = -\frac{(x - 5)^2}{x^2 + 5x + 25}$$

$$24. \frac{13x - 6 - 6x^2}{6x^2 - 13x + 6} = -\frac{(6x^2 - 13x + 6)}{6x^2 - 13x + 6} = -1$$

$$25. \frac{2x^3 - 2xy^2 + x^2 - y^2}{2xy^2 + y^2 - 2x^3 - x^2} = \frac{2x(x^2 - y^2) + (x^2 - y^2)}{y^2(2x + 1) - x^2(2x + 1)} = \frac{(2x + 1)(x^2 - y^2)}{(2x + 1)(y^2 - x^2)} = -\frac{(x^2 - y^2)}{(x^2 - y^2)} = -1$$

$$26. \frac{30x^2y - 45xy^2 - 20x^3}{8x^3 + 27y^3} = \frac{5x(6xy - 9y^2 - 4x^2)}{(2x + 3y)(4x^2 - 6xy + 9y^2)} = -\frac{5x(4x^2 - 6xy + 9y^2)}{(2x + 3y)(4x^2 - 6xy + 9y^2)} = -\frac{5x}{2x + 3y}$$

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$$\begin{aligned}
 27. \quad & \frac{n+1-n^3-n^2}{n^3-n-2n^2+2} \\
 &= \frac{(n+1)-n^2(n+1)}{n(n^2-1)-2(n^2-1)} \\
 &= \frac{(1-n^2)(n+1)}{(n-2)(n^2-1)} = \frac{(1-n^2)(n+1)}{(2-n)(1-n^2)} = \frac{n+1}{2-n}
 \end{aligned}$$

$$\begin{aligned}
 28. \quad & \frac{(x-2)^2(x^2+x-12)}{(2-x)(3-x)^2} \\
 &= \frac{(2-x)^2(x+4)(x-3)}{(2-x)(3-x)^2} \\
 &= \frac{(x-2)(x+4)(3-x)}{(3-x)^2} = \frac{(x-2)(x+4)}{3-x}
 \end{aligned}$$

$$\begin{aligned}
 29. \quad & \frac{5x^3-15x^2y}{90x^3y^2-10x^5} \\
 &= \frac{5x^2(x-3y)}{10x^3(9y^2-x^2)} \\
 &= \frac{(x-3y)}{2x(3y-x)(3y+x)} \\
 &= -\frac{x-3y}{2x(x-3y)(3y+x)} = \frac{-1}{2x(3y+x)}
 \end{aligned}$$

$$\begin{aligned}
 30. \quad & \frac{(x^2-1)(x^2-8x+16)}{(x^2-4x)(1-x^2)} \\
 &= \frac{(x+1)(x-1)(x-4)(x-4)}{x(x-4)(1+x)(1-x)} = \frac{(1-x)(4-x)}{x(1-x)} = \frac{4-x}{x}
 \end{aligned}$$

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Multiplicación de expresiones algebraicas

EJERCICIO 4

$$1. \frac{3}{2a} \cdot \frac{2a}{2a} = \frac{6a}{4a^2}$$

$$2. \frac{5}{9x^2} \cdot \frac{4a}{4a} = \frac{20a}{36ax^2}$$

$$3. \frac{m}{ab^2} \cdot \frac{2a}{2a} = \frac{2am}{2a^2b^2}$$

$$4. \frac{3x}{8y} \cdot \frac{3xy^2}{3xy^2} = \frac{9x^2y^2}{24xy^3}$$

$$5. \frac{4m}{5n^2} \cdot \frac{n}{n} = \frac{4mn}{5n^3}$$

$$6. \frac{2x+7}{5} \cdot \frac{3}{3} = \frac{6x+21}{15}$$

$$7. \frac{2x}{x-1} \cdot \frac{x}{x} = \frac{2x^2}{x^2-x}$$

$$8. \frac{a^2}{a+2} \cdot \frac{2a}{2a} = \frac{2a^3}{2a^2+4a}$$

$$9. \frac{3a}{a+b} \cdot \frac{a+b}{a+b} = \frac{3a^2+3ab}{a^2+2ab+b^2}$$

$$10. \frac{x-4}{x+3} \cdot \frac{x+2}{x+2} = \frac{x^2-2x-8}{x^2+5x+6}$$

$$11. \frac{2a}{x+a} \cdot \frac{a^2}{a^2} = \frac{2a^3}{a^2x+a^3}$$

$$12. \frac{x-y}{6} \cdot \frac{2}{2} = \frac{2x-2y}{12}$$

$$13. \frac{5x}{a-b} \cdot \frac{a+b}{a+b} = \frac{5ax+5bx}{a^2-b^2}$$

$$14. \frac{x-5}{a} \cdot \frac{3x}{3x} = \frac{3x^2-15x}{3ax}$$

$$15. \frac{5x}{2x+y} \cdot \frac{2x+y}{2x+y} = \frac{10x^2+5xy}{4x^2+4xy+y^2}$$

$$16. \frac{x+3}{x+1} \cdot \frac{x-3}{x-3} = \frac{x^2-9}{x^2-2x-3}$$

$$17. \frac{2}{a+1} \cdot \frac{a^2-a+1}{a^2-a+1} = \frac{2a^2-2a+2}{a^3+1}$$

$$18. \frac{x-2y}{3x} \cdot \frac{3xy}{3xy} = \frac{3x^2y-6xy^2}{9x^2y}$$

$$19. \frac{x-1}{x+1} \cdot \frac{x+1}{x+1} = \frac{x^2-1}{x^2+2x+1}$$

$$20. \frac{a+b}{7a^2} \cdot \frac{9ab}{9ab} = \frac{9a^2b+9ab^2}{63a^3b}$$

$$21. \frac{x+1}{x+5} \cdot \frac{x-2}{x-2} = \frac{x^2-x-2}{x^2+3x-10}$$

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Suma y resta de expresiones algebraicas

EJERCICIO 5

$$\begin{aligned} 1. \quad & a + \frac{4a}{a+2} \\ &= \frac{a(a+2)+4a}{a+2} \\ &= \frac{a^2+2a+4a}{a+2} = \frac{a^2+6a}{a+2} = \frac{a(a+6)}{a+2} \end{aligned}$$

$$\begin{aligned} 2. \quad & m - n - \frac{n^2}{m} \\ &= \frac{(m-n)m - n^2}{m} = \frac{m^2 - mn - n^2}{m} \end{aligned}$$

$$\begin{aligned} 3. \quad & x + 5 - \frac{3}{x-2} \\ &= \frac{(x+5)(x-2) - 3}{x-2} \\ &= \frac{x^2 + 3x - 10 - 3}{x-2} = \frac{x^2 + 3x - 13}{x-2} \end{aligned}$$

$$\begin{aligned} 4. \quad & a + \frac{ab}{a+b} \\ &= \frac{a(a+b)+ab}{a+b} \\ &= \frac{a^2+ab+ab}{a+b} = \frac{a^2+2ab}{a+b} \end{aligned}$$

$$\begin{aligned} 5. \quad & \frac{1-a^2}{a} + a - 3 \\ &= \frac{a(a-3)+1-a^2}{a} \\ &= \frac{a^2-3a+1-a^2}{a} = -\frac{3a-1}{a} \end{aligned}$$

$$\begin{aligned} 6. \quad & 1 - \frac{a+x}{a-x} \\ &= \frac{(a-x) - (a+x)}{a-x} \\ &= \frac{a-x-a-x}{a-x} = -\frac{2x}{a-x} \end{aligned}$$

$$\begin{aligned} 7. \quad & \frac{2a+x}{a+x} - 1 \\ &= \frac{2a+x - (a+x)}{a+x} \\ &= \frac{2a+x-a-x}{a+x} = \frac{a}{a+x} \end{aligned}$$

$$\begin{aligned} 8. \quad & x + 2 - \frac{3}{x-1} \\ &= \frac{(x+2)(x-1) - 3}{x-1} \\ &= \frac{x^2+x-2-3}{x-1} = \frac{x^2+x-5}{x-1} \end{aligned}$$

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$$\begin{aligned}
 9. \quad & x^2 - 3x - \frac{x^2 - 6x}{x+2} \\
 &= \frac{(x^2 - 3x)(x+2) - (x^2 - 6x)}{x+2} \\
 &= \frac{x^3 - x^2 - 6x - x^2 + 6x}{x+2} \\
 &= \frac{x^3 - 2x^2}{x+2} = \frac{x^2(x-2)}{x+2}
 \end{aligned}$$

$$\begin{aligned}
 10. \quad & x + y + \frac{x^2 - y^2}{x - y} \\
 &= \frac{(x+y)(x-y) + x^2 - y^2}{x-y} \\
 &= \frac{x^2 - y^2 + x^2 - y^2}{x-y} \\
 &= \frac{2x^2 - 2y^2}{x-y} \\
 &= \frac{2(x^2 - y^2)}{x-y} \\
 &= \frac{2(x+y)(x-y)}{x-y} \\
 &= 2x + 2y = 2(x+y)
 \end{aligned}$$

$$\begin{aligned}
 11. \quad & \frac{3mn}{m-n} + m - 2n \\
 &= \frac{(m-n)(m-2n) + 3mn}{m-n} \\
 &= \frac{m^2 - 3mn + 2n^2 + 3mn}{m-n} = \frac{m^2 + 2n^2}{m-n}
 \end{aligned}$$

$$\begin{aligned}
 12. \quad & 2a - 3x - \frac{5ax - 6x^2}{a+2x} \\
 &= \frac{(2a-3x)(a+2x) - (5ax-6x^2)}{a+2x} \\
 &= \frac{2a^2 + ax - 6x^2 - 5ax + 6x^2}{a+2x} \\
 &= \frac{2a^2 - 4ax}{a+2x}
 \end{aligned}$$

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$$\begin{aligned}
 13. \quad & m^2 - 2m + 4 - \frac{m^3}{m+2} \\
 &= \frac{(m^2 - 2m + 4)(m+2) - m^3}{m+2} \\
 &= \frac{m^3 + 8 - m^3}{m+2} = \frac{8}{m+2}
 \end{aligned}$$

$$\begin{aligned}
 14. \quad & x^2 - 5x - \frac{3x(x+2)}{x-2} \\
 &= \frac{(x^2 - 5x)(x-2) - 3x(x+2)}{x-2} \\
 &= \frac{x^3 - 7x^2 + 10x - 3x^2 - 6x}{x-2} \\
 &= \frac{x^3 - 10x^2 + 4x}{x-2}
 \end{aligned}$$

$$\begin{aligned}
 15. \quad & a^2 + 3ab - b^2 + \frac{7ab^2 - b^3}{2a-b} \\
 &= \frac{(a^2 + 3ab - b^2)(2a-b) + 7ab^2 - b^3}{2a-b} \\
 &= \frac{2a^3 + 5a^2b - 5ab^2 + b^3 + 7ab^2 - b^3}{2a-b} \\
 &= \frac{2a^3 + 5a^2b + 2ab^2}{2a-b}
 \end{aligned}$$

$$\begin{aligned}
 16. \quad & \frac{x^3 + 2}{x^2 - x + 1} - (x+1) \\
 &= \frac{(x^3 + 2) + (x^2 - x + 1)(-x-1)}{x^2 - x + 1} \\
 &= \frac{x^3 + 2 - x^3 - 1}{x^2 - x + 1} = \frac{1}{x^2 - x + 1}
 \end{aligned}$$

$$\begin{aligned}
 17. \quad & x + 3 - \frac{x^3 - 2x^2 + 1}{x^2 - 4x + 3} \\
 &= \frac{(x+3)(x^2 - 4x + 3) - (x^3 - 2x^2 + 1)}{x^2 - 4x + 3} \\
 &= \frac{x^3 - x^2 - 9x + 9 - x^3 + 2x^2 - 1}{x^2 - 4x + 3} \\
 &= \frac{x^2 - 9x + 8}{x^2 - 4x + 3} = \frac{(x-8)(x-1)}{(x-3)(x-1)} = \frac{x-8}{x-3}
 \end{aligned}$$

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$$\begin{aligned}
 18. \quad & 3a + \frac{3a^2b + 3ab^2}{a^2 - b^2} \\
 &= \frac{3a(a^2 - b^2) + 3a^2b + 3ab^2}{a^2 - b^2} \\
 &= \frac{3a^3 - 3ab^2 + 3a^2b + 3ab^2}{a^2 - b^2} \\
 &= \frac{3a^3 + 3a^2b}{a^2 - b^2} = \frac{3a^2(a+b)}{(a+b)(a-b)} \\
 &= \frac{3a^2}{a-b}
 \end{aligned}$$

$$\begin{aligned}
 19. \quad & x - 3 - \frac{x^3 - 27}{x^2 - 6x + 9} \\
 &= \frac{(x-3)(x^2 - 6x + 9) - (x^3 - 27)}{x^2 - 6x + 9} \\
 &= \frac{x^3 - 9x^2 + 27x - 27 - x^3 + 27}{x^2 - 6x + 9} \\
 &= \frac{-9x^2 + 27x}{(x-3)^2} = \frac{-9x(x-3)}{(x-3)^2} \\
 &= \frac{-9x}{x-3} = \frac{9x}{3-x}
 \end{aligned}$$

$$\begin{aligned}
 20. \quad & a^2 - 3a + 5 + \frac{2a^3 - 11a + 9}{a^2 + a - 2} \\
 &= \frac{(a^2 - 3a + 5)(a^2 + a - 2) + 2a^3 - 11a + 9}{a^2 + a - 2} \\
 &= \frac{a^4 - 2a^3 + 11a - 10 + 2a^3 - 11a + 9}{a^2 + a - 2} \\
 &= \frac{a^4 - 1}{(a+2)(a-1)} = \frac{(a^2-1)(a^2+1)}{(a+2)(a-1)} \\
 &= \frac{(a^2+1)(a+1)(a-1)}{(a+2)(a-1)} \\
 &= \frac{a^3 + a^2 + a + 1}{a+2}
 \end{aligned}$$

EJERCICIO 6

$$\begin{aligned}
 1. \quad & \frac{x-2}{4} + \frac{3x+2}{6} = \frac{3(x-2) + 2(3x+2)}{12} = \frac{3x-6+6x+4}{12} = \frac{9x-2}{12} \\
 2. \quad & \frac{2}{5a^2} + \frac{1}{3ab} = \frac{3b \cdot 2 + 5a}{15a^2b} = \frac{5a+6b}{15a^2b} \\
 3. \quad & \frac{a-2b}{15a} + \frac{b-a}{20b} = \frac{4b(a-2b) + 3a(b-a)}{60ab} \\
 &= \frac{4ab - 8b^2 + 3ab - 3a^2}{60ab} = \frac{-3a^2 + 7ab - 8b^2}{60ab} \\
 4. \quad & \frac{a+3b}{3ab} + \frac{a^2b-4ab^2}{5a^2b^2} = \frac{5ab(a+3b) + 3(a^2b-4ab^2)}{15a^2b^2} \\
 &= \frac{5a^2b + 15ab^2 + 3a^2b - 12ab^2}{15a^2b^2} \\
 &= \frac{8a^2b + 3ab^2}{15a^2b^2} = \frac{ab(8a+3b)}{15a^2b^2} = \frac{8a+3b}{15ab}
 \end{aligned}$$

$$5. \frac{a-1}{3} + \frac{2a}{6} + \frac{3a+4}{12}$$

$$= \frac{4(a-1) + 2 \cdot 2a + 3a + 4}{12} = \frac{4a - 4 + 4a + 3a + 4}{12} = \frac{11a}{12}$$

$$6. \frac{n}{m^2} + \frac{3}{mn} + \frac{2}{m} = \frac{n \cdot n + 3m + 2mn}{m^2 n} = \frac{3m + 2mn + n^2}{m^2 n}$$

$$7. \frac{1-x}{2x} + \frac{x+2}{x^2} + \frac{1}{3ax^2}$$

$$= \frac{3ax(1-x) + 6a(x+2) + 2}{6ax^2}$$

$$= \frac{3ax - 3ax^2 + 6ax - 12a + 2}{6ax^2} = \frac{9ax + 12a - 3ax^2 + 2}{6ax^2}$$

$$8. \frac{2a-3}{3a} + \frac{3x+2}{10x} + \frac{x-a}{5ax} = \frac{10x(2a-3) + 3a(3x+2) + 6(x-a)}{30ax}$$

$$= \frac{20ax - 30x + 9ax + 6a + 6x - 6a}{30ax}$$

$$= \frac{29ax - 24x}{30ax} = \frac{x(29a - 24)}{30ax} = \frac{29a - 24}{30a}$$

$$9. \frac{3}{5} + \frac{x+2}{2x} + \frac{x^2+2}{6x^2}$$

$$= \frac{6x^2 \cdot 3 + 15x(x+2) + 5(x^2+2)}{30x^2}$$

$$= \frac{18x^2 + 15x^2 + 30x + 5x^2 + 10}{30x^2}$$

$$= \frac{38x^2 + 30x + 10}{30x^2} = \frac{2(19x^2 + 15x + 5)}{30x^2} = \frac{19x^2 + 15x + 5}{15x^2}$$

$$10. \frac{x-y}{12} + \frac{2x+y}{15} + \frac{y-4x}{30}$$

$$= \frac{5(x-y) + 4(2x+y) + 2(y-4x)}{60}$$

$$= \frac{5x - 5y + 8x + 4y + 2y - 8x}{60} = \frac{5x + y}{60}$$

$$11. \frac{m-n}{mn} + \frac{n-a}{na} + \frac{2a-m}{am}$$

$$= \frac{a(m-n) + m(n-a) + n(2a-m)}{amn}$$

$$= \frac{am - an + mn - ma + 2an - mn}{amn} = \frac{an}{amn} = \frac{1}{m}$$

$$12. \frac{x+2}{3x} + \frac{x^2-2}{5x^2} + \frac{2-x^3}{9x^3}$$

$$= \frac{15x^2(x+2) + 9x(x^2-2) + 5(2-x^3)}{45x^3}$$

$$= \frac{15x^3 + 30x^2 + 9x^3 - 18x + 10 - 5x^3}{45x^3}$$

$$= \frac{19x^3 + 30x^2 - 18x + 10}{45x^3}$$

$$13. \frac{1}{ab} + \frac{b^2-a^2}{ab^3} + \frac{ab+b^2}{a^2b^2}$$

$$= \frac{ab^2 + a(b^2-a^2) + b(ab+b^2)}{a^2b^3}$$

$$= \frac{ab^2 + ab^2 - a^3 + ab^2 + b^3}{a^2b^3} = \frac{-a^3 + 3ab^2 + b^3}{a^2b^3}$$

$$14. \frac{a+3b}{ab} + \frac{2a-3m}{am} + \frac{3}{a}$$

$$= \frac{m(a+3b) + b(2a-3m) + 3bm}{abm}$$

$$= \frac{am + 3bm + 2ab - 3bm + 3bm}{abm} = \frac{am + 2ab + 3bm}{abm}$$

EJERCICIO 7

$$1. \frac{1}{a+1} + \frac{1}{a-1} = \frac{a-1+a+1}{a^2-1} = \frac{2a}{a^2-1}$$

$$2. \frac{2}{x+4} + \frac{1}{x-3} \\ = \frac{2(x-3)+x+4}{x^2+x-12} = \frac{2x-6+x+4}{x^2+x-12} = \frac{3x-2}{x^2+x-12}$$

$$3. \frac{3}{1-x} + \frac{6}{2x+5} \\ = \frac{3(2x+5)+6(1-x)}{(1-x)(2x+5)} = \frac{6x+15+6-6x}{(1-x)(2x+5)} = \frac{21}{(1-x)(2x+5)}$$

$$4. \frac{x}{x-y} + \frac{x}{x+y} \\ = \frac{x(x+y)+x(x-y)}{x^2-y^2} = \frac{x^2+xy+x^2-xy}{x^2-y^2} = \frac{2x^2}{x^2-y^2}$$

$$5. \frac{m+3}{m-3} + \frac{m+2}{m-2} \\ = \frac{(m-2)(m+3)+(m-3)(m+2)}{(m-3)(m-2)} \\ = \frac{m^2+m-6+m^2-m-6}{(m-3)(m-2)} = \frac{2m^2-12}{(m-3)(m-2)} = \frac{2(m^2-6)}{(m-3)(m-2)}$$

$$6. \frac{x+y}{x-y} + \frac{x-y}{x+y} = \frac{(x+y)(x+y)+(x-y)(x-y)}{x^2-y^2} \\ = \frac{(x+y)^2+(x-y)^2}{x^2-y^2} \\ = \frac{x^2+2xy+y^2+x^2-2xy+y^2}{x^2-y^2} = \frac{2x^2+2y^2}{x^2-y^2}$$

$$7. \frac{x}{x^2-1} + \frac{x+1}{(x-1)^2} = \frac{x(x-1)+(x+1)^2}{(x-1)^2(x+1)} \\ = \frac{x^2-x+x^2+2x+1}{(x-1)^2(x+1)} = \frac{2x^2+x+1}{(x-1)^2(x+1)}$$

$$8. \frac{2}{x-5} + \frac{3x}{x^2-25} = \frac{2(x+5)+3x}{x^2-25} = \frac{2x+10+3x}{x^2-25} = \frac{5x+10}{x^2-25}$$

$$9. \frac{1}{3x-2y} + \frac{x-y}{9x^2-4y^2} = \frac{3x+2y+x-y}{9x^2-4y^2} = \frac{4x+y}{9x^2-4y^2}$$

$$10. \frac{x+a}{x+3a} + \frac{3a^2-x^2}{x^2-9a^2} = \frac{(x-3a)(x+a)+3a^2-x^2}{x^2-9a^2} \\ = \frac{x^2+ax-3ax-3a^2+3a^2-x^2}{x^2-9a^2} \\ = \frac{-2ax}{x^2-9a^2} = \frac{2ax}{9a^2-x^2}$$

$$11. \frac{a}{1-a^2} + \frac{a}{1+a^2} = \frac{a(1+a^2) + a(1-a^2)}{(1-a^2)(1+a^2)} = \frac{a+a^3+a-a^3}{(1-a^2)(1+a^2)} = \frac{2a}{1-a^4}$$

$$12. \frac{2}{a^2-ab} + \frac{2}{ab+b^2} = \frac{2b(a+b) + 2a(a-b)}{ab(a^2-b^2)} = \frac{2ab+2b^2+2a^2-2ab}{ab(a^2-b^2)} = \frac{2a^2+2b^2}{ab(a^2-b^2)}$$

$$13. \frac{ab}{9a^2-b^2} + \frac{a}{3a+b} = \frac{ab+a(3a-b)}{9a^2-b^2} = \frac{ab+3a^2-ab}{9a^2-b^2} = \frac{3a^2}{9a^2-b^2}$$

$$14. \frac{1}{a^2-b^2} + \frac{1}{(a-b)^2} = \frac{a-b+a+b}{(a+b)(a-b)^2} = \frac{2a}{(a+b)(a-b)^2}$$

$$15. \frac{3}{x^2+y^2} + \frac{2}{(x+y)^2} = \frac{3(x+y)^2 + 2(x^2+y^2)}{(x^2+y^2)(x+y)^2} = \frac{3x^2+6xy+3y^2+2x^2+2y^2}{(x^2+y^2)(x+y)^2} = \frac{5x^2+6xy+5y^2}{(x^2+y^2)(x+y)^2}$$

$$16. \frac{x}{a^2-ax} + \frac{a+x}{ax} + \frac{a}{ax-x^2} = \frac{ax-x^2}{a^2-ax} = \frac{ax}{a(a-x)} = \frac{ax}{a^2-ax} = \frac{ax}{ax} = 1$$

$$17. \frac{3}{2x+4} + \frac{x-1}{2x-4} + \frac{x+8}{x^2-4} = \frac{3(x-2) + (x-1)(x+2) + 2(x+8)}{2(x^2-4)} = \frac{3x-6+x^2+x-2+2x+16}{2(x^2-4)} = \frac{x^2+6x+8}{2(x^2-4)} = \frac{(x+2)(x+4)}{2(x+2)(x-2)} = \frac{x+4}{2(x-2)}$$

$$18. \frac{1}{x+x^2} + \frac{1}{x-x^2} + \frac{x+3}{1-x^2} = \frac{1}{x(1+x)} + \frac{1}{x(1-x)} + \frac{x+3}{(1-x)(1+x)} = \frac{1-x+x+1+x+x(x+3)}{x(1-x^2)} = \frac{2+x^2+3x}{x(1-x^2)} = \frac{(x+1)(x+2)}{x(1-x)(1+x)} = \frac{x+2}{x(1-x)}$$

$$19. \frac{x-y}{x+y} + \frac{x+y}{x-y} + \frac{4xy}{x^2-y^2} = \frac{(x-y)(x-y) + (x+y)(x+y) + 4xy}{x^2-y^2} = \frac{x^2-2xy+y^2+x^2+2xy+y^2+4xy}{x^2-y^2} = \frac{2x^2+2y^2+4xy}{x^2-y^2} = \frac{2(x^2+2xy+y^2)}{x^2-y^2} = \frac{2(x+y)^2}{(x+y)(x-y)} = \frac{2(x+y)}{x-y}$$

$$20. \frac{1}{a-5} + \frac{a}{a^2-4a-5} + \frac{a+5}{a^2+2a+1} = \frac{1}{a-5} + \frac{a}{(a-5)(a+1)} + \frac{a+5}{(a+1)^2} = \frac{(a+1)^2 + a(a+1) + (a+5)(a-5)}{(a-5)(a+1)^2} = \frac{a^2+2a+1+a^2+a+a^2-25}{(a-5)(a+1)^2} = \frac{3a^2+3a-24}{(a-5)(a+1)^2} = \frac{3(a^2+a-8)}{(a-5)(a+1)^2}$$

$$21. \frac{3}{a} + \frac{2}{5a-3} + \frac{1-85a}{25a^2-9} = \frac{3(25a^2-9) + 2a(5a+3) + a(1-85a)}{a(25a^2-9)} = \frac{75a^2-27+10a^2+6a+a-85a^2}{a(25a^2-9)} = \frac{7a-27}{a(25a^2-9)}$$

$$22. \frac{x+1}{10} + \frac{x-3}{5x-10} + \frac{x-2}{2} = \frac{(x+1)(x-2) + 2(x-3) + 5(x-2)(x-2)}{10(x-2)} = \frac{x^2-x-2+2x-6+5x^2-20x+20}{10(x-2)} = \frac{6x^2-19x+12}{10(x-2)}$$

$$\begin{aligned}
 23. \quad & \frac{x+5}{x^2+x-12} + \frac{x+4}{x^2+2x-15} + \frac{x-3}{x^2+9x+20} \\
 & x^2+x-12=(x+4)(x-3) \\
 & x^2+2x-15=(x+5)(x-3) \\
 & x^2+9x+20=(x+5)(x+4) \\
 & mcm=(x+5)(x+4)(x-3) \\
 & = \frac{(x+5)(x+5)+(x+4)(x+4)+(x-3)(x-3)}{(x+5)(x+4)(x-3)} \\
 & = \frac{x^2+10x+25+x^2+8x+16+x^2-6x+9}{(x+5)(x+4)(x-3)} \\
 & = \frac{3x^2+12x+50}{(x+5)(x+4)(x-3)}
 \end{aligned}$$

$$\begin{aligned}
 24. \quad & \frac{1}{x-2} + \frac{1-2x^2}{x^3-8} + \frac{x}{x^2+2x+4} \\
 & x^3-8=(x-2)(x^2+2x+4) \quad mcm=x^3-8 \\
 & = \frac{x^2+2x+4+1-2x^2+x(x-2)}{x^3-8} \\
 & = \frac{-x^2+2x+5+x^2-2x}{x^3-8} = \frac{5}{x^3-8}
 \end{aligned}$$

$$\begin{aligned}
 25. \quad & \frac{2}{a+1} + \frac{a}{(a+1)^2} + \frac{a+1}{(a+1)^3} \quad mcm=(a+1)^3 \\
 & = \frac{2(a+1)^2+a(a+1)+a+1}{(a+1)^3} \\
 & = \frac{2a^2+4a+2+a^2+a+a+1}{(a+1)^3} \\
 & = \frac{3a^2+6a+3}{(a+1)^3} = \frac{3(a^2+2a+1)}{(a+1)^3} = \frac{3(a+1)^2}{(a+1)^3} = \frac{3}{a+1}
 \end{aligned}$$

$$\begin{aligned}
 26. \quad & \frac{2x}{3x^2+11x+6} + \frac{x+1}{x^2-9} + \frac{1}{3x+2} \\
 & 3x^2+11x+6=(x+3)(3x+2) \\
 & x^2-9=(x+3)(x-3) \\
 & 3x+2=3x+2 \quad mcm=(x^2-9)(3x+2) \\
 & = \frac{2x(x-3)+(x+1)(3x+2)+x^2-9}{(x^2-9)(3x+2)} \\
 & = \frac{2x^2-6x+3x^2+5x+2+x^2-9}{(x^2-9)(3x+2)} \\
 & = \frac{6x^2-x-7}{(x^2-9)(3x+2)}
 \end{aligned}$$

$$\begin{aligned}
 27. \quad & \frac{x^2-4}{x^3+1} + \frac{1}{x+1} + \frac{3}{x^2-x+1} \\
 & x^3+1=(x+1)(x^2-x+1) \quad mcm=x^3+1 \\
 & = \frac{x^2-4+x^2-x+1+3x+3}{x^3+1} \\
 & = \frac{2x^2+2x}{x^3+1} = \frac{2x(x+1)}{(x+1)(x^2-x+1)} = \frac{2x}{x^2-x+1}
 \end{aligned}$$

$$\begin{aligned}
 28. \quad & \frac{1}{x-1} + \frac{1}{(x-1)(x+2)} + \frac{x+1}{(x-1)(x+2)(x+3)} \\
 & mcm=(x-1)(x+2)(x+3) \\
 & = \frac{(x+2)(x+3)+x+3+x+1}{(x-1)(x+2)(x+3)} \\
 & = \frac{x^2+5x+6+2x+4}{(x-1)(x+2)(x+3)} = \frac{x^2+7x+10}{(x-1)(x+2)(x+3)} \\
 & = \frac{(x+5)(x+2)}{(x-1)(x+2)(x+3)} = \frac{x+5}{(x-1)(x+3)}
 \end{aligned}$$

$$\begin{aligned}
 29. \quad & \frac{x-2}{2x^2-5x-3} + \frac{x-3}{2x^2-3x-2} + \frac{2x-1}{x^2-5x+6} \\
 & 2x^2-5x-3=(x-3)(2x+1) \\
 & 2x^2-3x-2=(x-2)(2x+1) \\
 & x^2-5x+6=(x-3)(x-2) \\
 & mcm=(x-3)(x-2)(2x+1) \\
 & = \frac{(x-2)(x-2)+(x-3)(x-3)+(2x-1)(2x+1)}{(x-3)(x-2)(2x+1)} \\
 & = \frac{x^2-4x+4+x^2-6x+9+4x^2-1}{(x-3)(x-2)(2x+1)} \\
 & = \frac{6x^2-10x+12}{(x-3)(x-2)(2x+1)}
 \end{aligned}$$

$$\begin{aligned}
 30. \quad & \frac{a-2}{a-1} + \frac{a+3}{a+2} + \frac{a+1}{a-3} \quad mcm=(a-1)(a+2)(a-3) \\
 & = \frac{(a+2)(a-2)(a-3)+(a-1)(a+3)(a-3)+(a-1)(a+1)(a+2)}{(a-1)(a+2)(a-3)} \\
 & = \frac{a^3-3a^2-4a+12+a^3-a^2-9a+9+a^3+2a^2-a-2}{(a-1)(a+2)(a-3)} \\
 & = \frac{3a^3-2a^2-14a+19}{(a-1)(a+2)(a-3)}
 \end{aligned}$$

EJERCICIO 8

$$1. \frac{x-3}{4} - \frac{x+2}{8} = \frac{2(x-3)-(x+2)}{8} = \frac{2x-6-x-2}{8} = \frac{x-8}{8}$$

$$2. \frac{a+5b}{a^2} - \frac{b-3}{ab} = \frac{b(a+5b)-a(b-3)}{a^2b} = \frac{ab+5b^2-ab+3a}{a^2b} = \frac{3a+5b^2}{a^2b}$$

$$3. \frac{2}{3mn^2} - \frac{1}{2m^2n} = \frac{2(2m)-(3n)}{6m^2n^2} = \frac{4m-3n}{6m^2n^2}$$

$$4. \frac{a-3}{5ab} - \frac{4-3ab^2}{3a^2b^3} = \frac{3ab^2(a-3)-5(4-3ab^2)}{15a^2b^3} = \frac{3a^2b^2-9ab^2-20+15ab^2}{15a^2b^3} = \frac{3a^2b^2+6ab^2-20}{15a^2b^3}$$

$$5. \frac{2a+3}{4a} - \frac{a-2}{8a} = \frac{2(2a+3)-(a-2)}{8a} = \frac{4a+6-a+2}{8a} = \frac{3a+8}{8a}$$

$$6. \frac{y-2x}{20x} - \frac{x-3y}{24y} = \frac{6y(y-2x)-5x(x-3y)}{120xy} = \frac{6y^2-12xy-5x^2+15xy}{120xy} = \frac{6y^2+3xy-5x^2}{120xy}$$

$$7. \frac{x-1}{3} - \frac{x-2}{4} - \frac{x+3}{6} = \frac{4(x-1)-3(x-2)-2(x+3)}{12} = \frac{4x-4-3x+6-2x-6}{12} = \frac{-x-4}{12} = -\frac{x+4}{12}$$

$$8. \frac{3}{5} - \frac{2a+1}{10a} - \frac{4a^2+1}{20a^2} = \frac{4a^2(3)-2a(2a+1)-(4a^2+1)}{20a^2} = \frac{12a^2-4a^2-2a-4a^2-1}{20a^2} = \frac{4a^2-2a-1}{20a^2}$$

$$9. \frac{3}{5x} - \frac{x-1}{3x^2} - \frac{x^2+2x+3}{15x^3} = \frac{3x^2(3)-5x(x-1)-(x^2+2x+3)}{15x^3} = \frac{9x^2-5x^2+5x-x^2-2x-3}{15x^3} = \frac{3x^2+3x-3}{15x^3} = \frac{3(x^2+x-1)}{15x^3} = \frac{x^2+x-1}{5x^3}$$

$$10. \frac{1}{2a} - \frac{2+b}{3ab} - \frac{5}{6a^2b^3} = \frac{3ab^3-2ab^2(2+b)-5}{6a^2b^3} = \frac{3ab^3-4ab^2-2ab^3-5}{6a^2b^3} = \frac{ab^3-4ab^2-5}{6a^2b^3}$$

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EJERCICIO 9

$$1. \frac{1}{x-4} - \frac{1}{x-3} \quad mcm = (x-4)(x-3)$$
$$= \frac{x-3-(x-4)}{(x-4)(x-3)} = \frac{x-3-x+4}{(x-4)(x-3)} = \frac{1}{(x-4)(x-3)}$$

$$2. \frac{m-n}{m+n} - \frac{m+n}{m-n} \quad mcm = m^2 - n^2$$
$$= \frac{(m-n)(m-n) - (m+n)(m+n)}{m^2 - n^2}$$
$$= \frac{m^2 - 2mn + n^2 - m^2 - 2mn - n^2}{m^2 - n^2} = \frac{-4mn}{m^2 - n^2} = \frac{4mn}{n^2 - m^2}$$

$$3. \frac{1-x}{1+x} - \frac{1+x}{1-x} = \frac{(1-x)(1-x) - (1+x)(1+x)}{1-x^2}$$
$$= \frac{1-2x+x^2-1-2x-x^2}{1-x^2} = \frac{-4x}{1-x^2} = \frac{4x}{x^2-1}$$

$$4. \frac{a+b}{a^2+ab} - \frac{b-a}{ab+b^2}$$
$$a^2+ab = a(a+b) \quad ; \quad ab+b^2 = b(a+b)$$
$$mcm = ab(a+b)$$
$$= \frac{b(a+b) - a(b-a)}{ab(a+b)} = \frac{ab+b^2-ab+a^2}{ab(a+b)} = \frac{a^2+b^2}{ab(a+b)}$$

$$5. \frac{m+n}{m-n} - \frac{m^2+n^2}{m^2-n^2} = \frac{(m+n)(m+n) - (m^2+n^2)}{m^2-n^2}$$
$$= \frac{m^2+2mn+n^2-m^2-n^2}{m^2-n^2} = \frac{2mn}{m^2-n^2}$$

$$6. \frac{1}{x+x^2} - \frac{1}{x-x^2} \quad x+x^2 = x(x+1) \quad ; \quad x-x^2 = x(1-x)$$
$$mcm = x(1-x^2)$$
$$= \frac{1-x-(1+x)}{x(1-x^2)} = \frac{1-x-1-x}{x(1-x^2)} = \frac{-2x}{x(1-x^2)} = \frac{-2}{1-x^2} = \frac{2}{x^2-1}$$

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$$7. \frac{a+x}{(a-x)^2} - \frac{x}{a^2-x^2}$$

$$= \frac{(a+x)(a+x) - x(a-x)}{(a-x)^2(a+x)}$$

$$= \frac{a^2+2ax+x^2-ax+x^2}{(a-x)^2(a+x)} = \frac{a^2+ax+2x^2}{(a-x)^2(a+x)}$$

$$8. \frac{a+1}{6a+3} - \frac{1}{12a+6}$$

$$6a+3=3(2a+1) \quad ; \quad 12a+6=6(2a+1)$$

$$mcm=6(2a+1)$$

$$= \frac{2(a+1)-1}{6(2a+1)} = \frac{2a+2-1}{6(2a+1)} = \frac{2a+1}{6(2a+1)} = \frac{1}{6}$$

$$9. \frac{a-4}{a^2-6a+9} - \frac{a+3}{a^2+a-12}$$

$$a^2-6a+9=(a-3)^2 \quad ; \quad a^2+a-12=(a+4)(a-3)$$

$$mcm=(a-3)^2(a+4)$$

$$= \frac{(a+4)(a-4) - (a-3)(a+3)}{(a+4)(a-3)^2}$$

$$= \frac{a^2-16-a^2+9}{(a+4)(a-3)^2} = \frac{-7}{(a+4)(a-3)^2}$$

$$10. \frac{a^2+4ab-3b^2}{a^2-9b^2} - \frac{b}{a+3b} \quad mcm=a^2-9b^2$$

$$= \frac{a^2+4ab-3b^2-b(a-3b)}{a^2-9b^2}$$

$$= \frac{a^2+4ab-3b^2-ab+3b^2}{a^2-9b^2}$$

$$= \frac{a^2+3ab}{a^2-9b^2} = \frac{a(a+3b)}{(a+3b)(a-3b)} = \frac{a}{a-3b}$$

$$11. \frac{x}{x^2-1} - \frac{x+1}{(x-1)^2} \quad mcm=(x-1)^2(x+1)$$

$$= \frac{x(x-1) - (x+1)(x+1)}{(x+1)(x-1)^2}$$

$$= \frac{x^2-x-x^2-2x-1}{(x+1)(x-1)^2} = \frac{-3x-1}{(x+1)(x-1)^2} = \frac{3x+1}{(x+1)(x-1)^2}$$

$$12. \frac{1}{a^3-b^3} - \frac{1}{(a-b)^3} \quad mcm=(a-b)^3(a^2+ab+b^2)$$

$$= \frac{(a-b)^2 - (a^2+ab+b^2)}{(a-b)^3(a^2+ab+b^2)}$$

$$= \frac{a^2-2ab+b^2-a^2-ab-b^2}{(a-b)^3(a^2+ab+b^2)} = \frac{-3ab}{(a-b)^3(a^2+ab+b^2)}$$

$$13. \frac{x+3}{6x^2+x-2} - \frac{1}{4x^2-4x+1}$$

$$6x^2+x-2=(3x+2)(2x-1)$$

$$4x^2-4x+1=(2x-1)^2 \quad mcm=(2x-1)^2(3x+2)$$

$$= \frac{(2x-1)(x+3) - (3x+2)}{(3x+2)(2x-1)^2}$$

$$= \frac{2x^2+5x-3-3x-2}{(3x+2)(2x-1)^2} = \frac{2x^2+2x-5}{(3x+2)(2x-1)^2}$$

$$14. \frac{x-1}{4x+4} - \frac{x+2}{8x-8} \quad mcm=8(x^2-1)$$

$$= \frac{2(x-1)(x-1) - (x+2)(x+1)}{8(x^2-1)}$$

$$= \frac{2x^2-4x+2-x^2-3x-2}{8(x^2-1)} = \frac{x^2-7x}{8(x^2-1)}$$

$$15. \frac{x}{xy-y^2} - \frac{1}{y} \quad mcm=y(x-y)$$

$$= \frac{x-(x-y)}{y(x-y)} = \frac{x-x+y}{y(x-y)} = \frac{y}{y(x-y)} = \frac{1}{x-y}$$

$$16. \frac{b}{a^2-b^2} - \frac{b}{a^2+ab} \quad mcm=a(a^2-b^2)$$

$$= \frac{ab-b(a-b)}{a(a^2-b^2)} = \frac{ab-ab+b^2}{a(a^2-b^2)} = \frac{b^2}{a(a^2-b^2)}$$

$$17. \frac{2a-3}{6a+9} - \frac{a-1}{4a^2+12a+9}$$

$$6a+9=3(2a+3) \quad ; \quad 4a^2+12a+9=(2a+3)^2$$

$$mcm=3(2a+3)^2$$

$$= \frac{(2a+3)(2a-3) - 3(a-1)}{3(2a+3)^2}$$

$$= \frac{4a^2-9-3a+3}{3(2a+3)^2} = \frac{4a^2-3a-6}{3(2a+3)^2}$$

$$18. \frac{x+1}{x^2+x+1} - \frac{x-1}{x^2-x+1} \quad mcm=(x^2+x+1)(x^2-x+1)$$

$$= \frac{x+1-(x-1)}{(x^2+x+1)(x^2-x+1)}$$

$$= \frac{x+1-x+1}{(x^2+x+1)(x^2-x+1)} = \frac{2}{(x^2+x+1)(x^2-x+1)}$$

19. $\frac{a-1}{a^2+a} - \frac{1}{2a-2} - \frac{1}{2a+2}$ $mcm = 2a(a^2-1)$

$$= \frac{2(a-1)(a-1) - a(a+1) - a(a-1)}{2a(a^2-1)}$$

$$= \frac{2(a^2-2a+1) - a^2 - a - a^2 + a}{2a(a^2-1)}$$

$$= \frac{2a^2 - 4a + 2 - 2a^2}{2a(a^2-1)}$$

$$= -\frac{4a-2}{2a(a^2-1)}$$

$$= -\frac{2(2a-1)}{2a(a^2-1)} = -\frac{2a-1}{a(a^2-1)} = \frac{1-2a}{a(a^2-1)}$$

20. $\frac{1}{4a+4} - \frac{1}{8a-8} - \frac{1}{12a^2+12}$

$$mcm = 24(a^2+1)(a^2-1)$$

$$= \frac{6(a^2+1)(a-1) - 3(a^2+1)(a+1) - 2(a^2-1)}{24(a^2+1)(a^2-1)}$$

$$= \frac{6(a^3 - a^2 + a - 1) - 3(a^3 + a^2 + a + 1) - 2a^2 + 2}{24(a^2+1)(a^2-1)}$$

$$= \frac{6a^3 - 6a^2 + 6a - 6 - 3a^3 - 3a^2 - 3a - 3 - 2a^2 + 2}{24(a^2+1)(a^2-1)}$$

$$= \frac{3a^3 - 11a^2 + 3a - 7}{24(a^4-1)}$$

21. $\frac{y}{x^2-xy} - \frac{1}{x} - \frac{1}{x-y}$ $mcm = x(x-y)$

$$= \frac{y(x-y) - x(x-y) - x(x-y)}{x(x-y)}$$

$$= \frac{y-x+y-x}{x(x-y)}$$

$$= \frac{2y-2x}{x(x-y)} = \frac{2(y-x)}{x(x-y)} = -\frac{2(x-y)}{x(x-y)} = -\frac{2}{x}$$

22. $\frac{a}{a^2+ab} - \frac{1}{a} - \frac{1}{a+b}$ $mcm = a(a+b)$

$$= \frac{a-(a+b)-a}{a(a+b)} = \frac{-a-b}{a(a+b)} = -\frac{a+b}{a(a+b)} = -\frac{1}{a}$$

23. $\frac{1}{x^2-xy} - \frac{1}{x^2+xy} - \frac{2y}{x^3-xy^2}$ $mcm = x(x^2-y^2)$

$$= \frac{x+y-(x-y)-2y}{x(x^2-y^2)} = \frac{x-y-x+y}{x(x^2-y^2)} = \frac{0}{x(x^2-y^2)} = 0$$

24. $\frac{x}{x^2+x-2} - \frac{3}{x^2+2x-3} - \frac{x}{x^2+5x+6}$

$$x^2+x-2 = (x+2)(x-1)$$

$$x^2+2x-3 = (x+3)(x-1)$$

$$x^2+5x+6 = (x+3)(x+2)$$

$$mcm = (x+2)(x-1)(x+3)$$

$$= \frac{x(x+3) - 3(x+2) - x(x-1)}{(x+2)(x-1)(x+3)}$$

$$= \frac{x^2+3x-3x-6-x^2+x}{(x+2)(x-1)(x+3)} = \frac{x-6}{(x+2)(x-1)(x+3)}$$

25. $\frac{3}{x^2+x+1} - \frac{x+2}{(x-1)^2} - \frac{1-9x}{(x^3-1)(x-1)}$ $mcm = (x^3-1)(x-1)$

$$= \frac{3(x-1)^2 - (x+2)(x^2+x+1) - (1-9x)}{(x^3-1)(x-1)}$$

$$= \frac{3x^2-6x+3-x^3-x^2-x-2x^2-2x-2-1+9x}{(x^3-1)(x-1)}$$

$$= -\frac{x^3}{(x^3-1)(x-1)}$$

26. $\frac{a^2+b^2}{a^3-b^3} - \frac{a+b}{2a^2+2ab+2b^2} - \frac{1}{2a-2b}$

$$a^3-b^3 = (a-b)(a^2+ab+b^2)$$

$$2a^2+2ab+2b^2 = 2(a^2+ab+b^2)$$

$$2a-2b = 2(a-b)$$

$$mcm = 2(a-b)(a^2+ab+b^2)$$

$$= \frac{2(a^2+b^2) - (a+b)(a-b) - (a^2+ab+b^2)}{2(a-b)(a^2+ab+b^2)}$$

$$= \frac{2a^2+2b^2-a^2+b^2-a^2-ab-b^2}{2(a-b)(a^2+ab+b^2)}$$

$$= \frac{2b^2-ab}{2(a-b)(a^2+ab+b^2)} = \frac{2b^2-ab}{2(a^3-b^3)}$$

27. $\frac{3a}{2a^2-2a-4} - \frac{a-1}{4a^2+8a-32} - \frac{10a-1}{8a^2+40a+32}$

$$2a^2-2a-4 = 2(a^2-a-2) = 2(a-2)(a+1)$$

$$4a^2+8a-32 = 4(a^2+2a-8) = 4(a+4)(a-2)$$

$$8a^2+40a+32 = 8(a^2+5a+4) = 8(a+4)(a+1)$$

$$mcm = 8(a+4)(a-2)(a+1)$$

$$= \frac{12a(a+4) - 2(a-1)(a+1) - (10a-1)(a-2)}{8(a+4)(a-2)(a+1)}$$

$$= \frac{12a^2+48a-2a^2+2-10a^2+20a+a-2}{8(a+4)(a-2)(a+1)}$$

$$= \frac{69a}{8(a+4)(a-2)(a+1)}$$

$$\begin{aligned}
 28. \quad & \frac{1}{4a-12x} - \frac{a^2+9x^2}{a^3-27x^3} - \frac{a}{2(a^2+3ax+9a^2)} \\
 & 4a-12x=4(a-3x) \\
 & a^3-27x^3=(a-3x)(a^2+3ax+9a^2) \\
 & 2(a^2+3ax+9a^2)=2(a^2+3ax+9a^2) \\
 & mcm=4(a^3-27x^3) \\
 & = \frac{a^2+3ax+9x^2-4(a^2+9x^2)-2a(a-3x)}{4(a^3-27x^3)} \\
 & = \frac{a^2+3ax+9x^2-4a^2-36x^2-2a^2+6ax}{4(a^3-27x^3)} \\
 & = \frac{-5a^2+9ax-27x^2}{4(a^3-27x^3)} = -\frac{5a^2-9ax+27x^2}{4(a^3-27x^3)}
 \end{aligned}$$

$$\begin{aligned}
 29. \quad & \frac{2a^2-3}{10a+10} - \frac{a+1}{50} - \frac{9a^2-14}{50a+50} \\
 & 10a+10=10(a+1) \\
 & 50=50 \\
 & 50a+50=50(a+1) \\
 & mcm=50(a+1) \\
 & = \frac{5(2a^2-3)-(a+1)(a+1)-(9a^2-14)}{50(a+1)} \\
 & = \frac{10a^2-15-a^2-2a-1-9a^2+14}{50(a+1)} \\
 & = \frac{-2a-2}{50(a+1)} = -\frac{2(a+1)}{50(a+1)} = -\frac{1}{25}
 \end{aligned}$$

EJERCICIO 10

$$\begin{aligned}
 1. \quad & \frac{2}{x-3} + \frac{3}{x+2} - \frac{4x-7}{x^2-x-6} \quad mcm=(x-3)(x+2) \\
 & = \frac{2(x+2)+3(x-3)-(4x-7)}{(x-3)(x+2)} \\
 & = \frac{2x+4+3x-9-4x+7}{(x-3)(x+2)} = \frac{x+2}{(x-3)(x+2)} = \frac{1}{x-3}
 \end{aligned}$$

$$\begin{aligned}
 5. \quad & \frac{a-b}{a^2+ab} + \frac{a+b}{ab} - \frac{a}{ab+b^2} \quad mcm=ab(a+b) \\
 & = \frac{b(a-b)+(a+b)(a+b)-a^2}{ab(a+b)} \\
 & = \frac{ab-b^2+a^2+2ab+b^2-a^2}{ab(a+b)} = \frac{3ab}{ab(a+b)} = \frac{3}{a+b}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & \frac{a}{3a+6} - \frac{1}{6a+12} + \frac{a+12}{12a+24} \quad mcm=12(a+2) \\
 & = \frac{4a-2+a+12}{12(a+2)} = \frac{5a+10}{12(a+2)} = \frac{5(a+2)}{12(a+2)} = \frac{5}{12}
 \end{aligned}$$

$$\begin{aligned}
 6. \quad & \frac{x-y}{x+y} - \frac{x+y}{x-y} + \frac{4x^2}{x^2-y^2} \quad mcm=x^2-y^2 \\
 & = \frac{(x-y)(x-y)-(x+y)(x+y)+4x^2}{x^2-y^2}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad & \frac{x}{x^2+1} + \frac{1}{3x} - \frac{1}{x^2} \quad mcm=3x^2(x^2+1) \\
 & = \frac{3x^3+x(x^2+1)-3(x^2+1)}{3x^2(x^2+1)} \\
 & = \frac{3x^3+x^3+x-3x^2-3}{3x^2(x^2+1)} = \frac{4x^3+x-3x^2-3}{3x^2(x^2+1)}
 \end{aligned}$$

$$\begin{aligned}
 & = \frac{x^2-2xy+y^2-x^2-2xy-y^2+4x^2}{x^2-y^2} \\
 & = \frac{4x^2-4xy}{x^2-y^2} = \frac{4x(x-y)}{(x+y)(x-y)} = \frac{4x}{x+y}
 \end{aligned}$$

$$\begin{aligned}
 4. \quad & \frac{a+3}{a^2-1} + \frac{a-1}{2a+2} + \frac{a-4}{4a-4} \quad mcm=4(a^2-1) \\
 & = \frac{4(a+3)+2(a-1)(a-1)+(a-4)(a+1)}{4(a^2-1)} \\
 & = \frac{4a+12+2a^2-4a+2+a^2-3a-4}{4(a^2-1)} = \frac{3a^2-3a+10}{4(a^2-1)}
 \end{aligned}$$

$$\begin{aligned}
 7. \quad & \frac{x}{a^2-ax} + \frac{1}{a} + \frac{1}{x} \quad mcm=ax(a-x) \\
 & = \frac{x^2+x(a-x)+a(a-x)}{ax(a-x)} \\
 & = \frac{x^2+ax-x^2+a^2-ax}{ax(a-x)} = \frac{a^2}{ax(a-x)} = \frac{a}{x(a-x)}
 \end{aligned}$$

$$\begin{aligned}
 8. \quad & \frac{x+1}{x^2-x-20} - \frac{x+4}{x^2-4x-5} + \frac{x+5}{x^2+5x+4} \\
 & x^2-x-20=(x-5)(x+4) \\
 & x^2-4x-5=(x-5)(x+1) \\
 & x^2+5x+4=(x+4)(x+1) \\
 & mcm=(x-5)(x+4)(x+1) \\
 & = \frac{(x+1)(x+1)-(x+4)(x+4)+(x-5)(x+5)}{(x-5)(x+4)(x+1)} \\
 & = \frac{x^2+2x+1-x^2-8x-16+x^2-25}{(x-5)(x+4)(x+1)} \\
 & = \frac{x^2-6x-40}{(x-5)(x+4)(x+1)} \\
 & = \frac{(x-10)(x+4)}{(x-5)(x+4)(x+1)} = \frac{x-10}{(x-5)(x+1)}
 \end{aligned}$$

$$\begin{aligned}
 9. \quad & \frac{2x+1}{12x+8} - \frac{x^2}{6x^2+x-2} + \frac{2x}{16x-8} \\
 & 12x+8=4(3x+2) \\
 & 6x^2+x-2=(3x+2)(2x-1) \\
 & 16x-8=8(2x-1) \\
 & mcm=8(3x+2)(2x-1) \\
 & = \frac{2(2x+1)(2x-1)-8x^2+2x(3x+2)}{8(3x+2)(2x-1)} \\
 & = \frac{8x^2-2-8x^2+6x^2+4x}{8(3x+2)(2x-1)} \\
 & = \frac{6x^2+4x-2}{8(3x+2)(2x-1)} \\
 & = \frac{2(3x^2+2x-1)}{8(3x+2)(2x-1)} = \frac{3x^2+2x-1}{4(3x+2)(2x-1)}
 \end{aligned}$$

$$\begin{aligned}
 10. \quad & \frac{1}{ax} - \frac{1}{a^2+ax} + \frac{1}{a+x} \quad mcm=ax(a+x) \\
 & = \frac{a+x-x+ax}{ax(a+x)} = \frac{ax+a}{ax(a+x)} = \frac{a(x+1)}{ax(a+x)} = \frac{x+1}{x(a+x)}
 \end{aligned}$$

$$\begin{aligned}
 11. \quad & \frac{1}{x+y} - \frac{1}{x-y} + \frac{2y}{x^2+y^2} \quad mcm=(x^2-y^2)(x^2+y^2) \\
 & = \frac{(x-y)(x^2+y^2)-(x+y)(x^2+y^2)+2y(x^2-y^2)}{(x^2-y^2)(x^2+y^2)} \\
 & = \frac{x^3+xy^2-x^2y-y^3-x^3-xy^2-x^2y-y^3+2x^2y-2y^3}{(x^2-y^2)(x^2+y^2)} \\
 & = \frac{-4y^3}{x^4-y^4} = \frac{4y^3}{y^4-x^4}
 \end{aligned}$$

$$\begin{aligned}
 12. \quad & \frac{a-1}{3a+3} - \frac{a-2}{6a-6} + \frac{a^2+2a-6}{9a^2-9} \quad mcm=18(a^2-1) \\
 & = \frac{6(a-1)(a-1)-3(a-2)(a+1)+2(a^2+2a-6)}{18(a^2-1)} \\
 & = \frac{6a^2-12a+6-3a^2+3a+6+2a^2+4a-12}{18(a^2-1)} \\
 & = \frac{5a^2-5a}{18(a^2-1)} = \frac{5a(a-1)}{18(a+1)(a-1)} = \frac{5a}{18(a+1)}
 \end{aligned}$$

$$\begin{aligned}
 13. \quad & \frac{1}{a^2+2a-24} + \frac{2}{a^2-2a-8} - \frac{3}{a^2+8a+12} \\
 & a^2+2a-24=(a+6)(a-4) \\
 & a^2-2a-8=(a-4)(a+2) \\
 & a^2+8a+12=(a+6)(a+2) \\
 & mcm=(a+6)(a+2)(a-4) \\
 & = \frac{a+2+2(a+6)-3(a-4)}{(a+6)(a+2)(a-4)} \\
 & = \frac{a+2+2a+12-3a+12}{(a+6)(a+2)(a-4)} = \frac{26}{(a+6)(a+2)(a-4)}
 \end{aligned}$$

$$\begin{aligned}
 14. \quad & \frac{x+y}{xy} - \frac{x+2y}{xy+y^2} - \frac{y}{x^2+xy} \quad mcm=xy(x+y) \\
 & = \frac{(x+y)(x+y)-x(x+2y)-y^2}{xy(x+y)} \\
 & = \frac{x^2+2xy+y^2-x^2-2xy-y^2}{xy(x+y)} = \frac{0}{xy(x+y)} = 0
 \end{aligned}$$

$$\begin{aligned}
 15. \quad & \frac{a^3}{a^3+1} + \frac{a+3}{a^2-a+1} - \frac{a-1}{a+1} \quad mcm=a^3+1 \\
 & = \frac{a^3+(a+1)(a+3)-(a-1)(a^2-a+1)}{a^3+1} \\
 & = \frac{a^3+a^2+4a+3-a^3+a^2-a+a^2-a+1}{a^3+1} = \frac{3a^2+2a+4}{a^3+1}
 \end{aligned}$$

$$\begin{aligned}
 16. \quad & \frac{1}{x-1} + \frac{2x}{x^2-1} - \frac{3x^2}{x^3-1} \quad mcm=(x^2-1)(x^2+x+1) \\
 & = \frac{(x+1)(x^2+x+1)+2x(x^2+x+1)-3x^2(x+1)}{(x^2-1)(x^2+x+1)} \\
 & = \frac{x^3+x^2+x+x^2+x+1+2x^3+2x^2+2x-3x^3-3x^2}{(x^2-1)(x^2+x+1)} \\
 & = \frac{x^2+4x+1}{(x^2-1)(x^2+x+1)}
 \end{aligned}$$

$$17. \frac{a+b}{a^2-ab+b^2} - \frac{1}{a+b} + \frac{3a^2}{a^3+b^3} \quad mcm=a^3+b^3$$

$$= \frac{(a+b)(a+b) - (a^2-ab+b^2) + 3a^2}{a^3+b^3}$$

$$= \frac{a^2+2ab+b^2 - a^2+ab-b^2 + 3a^2}{a^3+b^3}$$

$$= \frac{3a^2+3ab}{a^3+b^3} = \frac{3a(a+b)}{(a+b)(a^2-ab+b^2)} = \frac{3a}{a^2-ab+b^2}$$

$$18. \frac{2}{x-2} + \frac{2x+3}{x^2+2x+4} - \frac{6x+12}{x^3-8} \quad mcm=x^3-8$$

$$= \frac{2(x^2+2x+4) + (2x+3)(x-2) - (6x+12)}{x^3-8}$$

$$= \frac{2x^2+4x+8+2x^2-4x+3x-6-6x-12}{x^3-8}$$

$$= \frac{4x^2-3x-10}{x^3-8} = \frac{(x-2)(4x+5)}{x^3-8} = \frac{4x+5}{x^2+2x+4}$$

$$19. \frac{3x+2}{x^2+3x-10} - \frac{5x+1}{x^2+4x-5} + \frac{4x-1}{x^2-3x+2}$$

$$x^2+3x-10=(x+5)(x-2); \quad x^2+4x-5=(x+5)(x-1)$$

$$x^2-3x+2=(x-2)(x-1); \quad mcm=(x+5)(x-2)(x-1)$$

$$= \frac{(3x+2)(x-1) - (5x+1)(x-2) + (4x-1)(x+5)}{(x+5)(x-2)(x-1)}$$

$$= \frac{3x^2-3x+2x-2-5x^2+10x-x+2+4x^2+20x-x-5}{(x+5)(x-2)(x-1)}$$

$$= \frac{2x^2+27x-5}{(x+5)(x-2)(x-1)}$$

$$20. \frac{1}{(n+1)^2} + \frac{1}{n-1} - \frac{1}{(n-1)^3} - \frac{1}{n} \quad mcm=n(n-1)^3$$

$$= \frac{n(n-1) + n(n-1)^2 - n - (n-1)^3}{n(n-1)^3}$$

$$= \frac{n^2-n+n^3-2n^2+n-n-n^3+3n^2-3n+1}{n(n-1)^3} = \frac{2n^2-4n+1}{n(n-1)^3}$$

$$21. \frac{1}{a^2+5} - \frac{a^2-5}{(a^2+5)^2} + \frac{a^2+5}{a^4-25} \quad mcm=(a^4-25)(a^2+5)$$

$$= \frac{a^4-25 - (a^2-5)(a^2-5) + (a^2+5)(a^2+5)}{(a^4-25)(a^2+5)}$$

$$= \frac{a^4-25-a^4+10a^2-25+a^4+10a^2+25}{(a^4-25)(a^2+5)} = \frac{a^4+20a^2-25}{(a^4-25)(a^2+5)}$$

$$22. \frac{1-x^2}{9-x^2} - \frac{x^2}{9+6x+x^2} - \frac{6x}{9-6x+x^2}$$

$$9-x^2=(3+x)(3-x); \quad 9+6x+x^2=(3+x)^2$$

$$9-6x+x^2=(3-x)^2; \quad mcm=(3+x)^2(3-x)^2$$

$$= \frac{(1-x^2)(9-x^2) - x^2(3-x)^2 - 6x(3+x)^2}{(3+x)^2(3-x)^2}$$

$$= \frac{9-x^2-9x^2+x^4-9x^2+6x^3-x^4-54x-36x^2-6x^3}{(3+x)^2(3-x)^2}$$

$$= \frac{9-54x-55x^2}{(3+x)^2(3-x)^2}$$

$$23. \frac{x}{2x+2} - \frac{x+1}{3x-3} + \frac{x-1}{6x+6} - \frac{5}{18x-18} \quad mcm=18(x^2-1)$$

$$= \frac{9x(x-1) - 6(x+1)(x+1) + 3(x-1)(x-1) - 5(x+1)}{18(x^2-1)}$$

$$= \frac{9x^2-9x-6x^2-12x-6+3x^2-6x+3-5x-5}{18(x^2-1)}$$

$$= \frac{6x^2-32x-8}{18(x^2-1)} = \frac{2(3x^2-16x-4)}{18(x^2-1)} = \frac{3x^2-16x-4}{9(x^2-1)}$$

$$24. \frac{a+2}{2a+2} - \frac{7a}{8a^2-8} - \frac{a-3}{4a-4} \quad mcm=8(a^2-1)$$

$$= \frac{4(a+2)(a-1) - 7a - 2(a-3)(a+1)}{8(a^2-1)}$$

$$= \frac{4a^2+4a-8-7a-2a^2+4a+6}{8(a^2-1)} = \frac{2a^2+a-2}{8(a^2-1)}$$

$$25. \frac{a-3}{20a+10} + \frac{2a+5}{40a+20} - \frac{4a-1}{60a+30} \quad mcm=120(2a+1)$$

$$= \frac{12(a-3) + 6(2a+5) - 4(4a-1)}{120(2a+1)}$$

$$= \frac{12a-36+12a+30-16a+4}{120(2a+1)}$$

$$= \frac{8a-2}{120(2a+1)} = \frac{2(4a-1)}{120(2a+1)} = \frac{4a-1}{60(2a+1)}$$

$$26. \frac{2}{2x^2+5x+3} - \frac{1}{2x^2-x-6} + \frac{3}{x^2-x-2}$$

$$2x^2+5x+3=(2x+3)(x+1)$$

$$2x^2-x-6=(2x+3)(x-2)$$

$$x^2-x-2=(x-2)(x+1)$$

$$mcm=(2x+3)(x+1)(x-2)$$

$$= \frac{2(x-2) - (x+1) + 3(2x+3)}{(2x+3)(x+1)(x-2)}$$

$$= \frac{2x-4-x-1+6x+9}{(2x+3)(x+1)(x-2)} = \frac{7x+4}{(2x+3)(x+1)(x-2)}$$

$$27. \frac{a-1}{a-2} - \frac{a-2}{a+3} + \frac{1}{a-1} \quad mcm=(a-2)(a+3)(a-1)$$

$$= \frac{(a-1)^2(a+3) - (a-2)^2(a-1) + (a-2)(a+3)}{(a-2)(a+3)(a-1)}$$

$$= \frac{(a^2-2a+1)(a+3) - (a^2-4a+4)(a-1) + a^2+a-6}{(a-2)(a+3)(a-1)}$$

$$= \frac{a^3+3a^2-2a^2-6a+a+3-a^3+a^2+4a^2-4a-4a+4+a^2+a-6}{(a-2)(a+3)(a-1)}$$

$$= \frac{7a^2-12a+1}{(a-2)(a+3)(a-1)}$$

$$28. \frac{2+3a}{2-3a} - \frac{2-3a}{2+3a} - \frac{a}{(2-3a)^2} \quad mcm=(4-9a^2)(2-3a)$$

$$= \frac{(2+3a)(4-9a^2) - (2-3a)^3 - a(2+3a)}{(4-9a^2)(2-3a)}$$

$$= \frac{8-18a^2+12a-27a^3-8+36a-54a^2+27a^3-2a-3a^2}{(4-9a^2)(2-3a)}$$

$$= \frac{46a-75a^2}{(4-9a^2)(2-3a)}$$

$$29. \frac{1}{5+5a} + \frac{1}{5-5a} - \frac{1}{10+10a^2} \quad mcm=10(1+a^2)(1-a^2)$$

$$= \frac{2(1+a^2)(1-a) + 2(1+a^2)(1+a) - (1-a^2)}{10(1+a^2)(1-a^2)}$$

$$= \frac{2-2a+2a^2-2a^3+2+2a+2a^2+2a^3-1+a^2}{10(1+a^2)(1-a^2)}$$

$$= \frac{5a^2+3}{10(1-a^4)}$$

$$30. \frac{1}{3-3x} - \frac{1}{3+3x} + \frac{x}{6+6x^2} - \frac{x}{2-2x^2}$$

$$3-3x=3(1-x) \quad ; \quad 3+3x=3(1+x)$$

$$6+6x^2=6(1+x^2) \quad ; \quad 2-2x^2=2(1-x^2)$$

$$mcm=6(1+x^2)(1-x^2)=6(1-x^4)$$

$$= \frac{2(1+x^2)(1+x) - 2(1+x^2)(1-x) + x(1-x^2) - 3x(1+x^2)}{6(1-x^4)}$$

$$= \frac{2+2x+2x^2+2x^3-2+2x-2x^2+2x^3+x-x^3-3x-3x^3}{6(1-x^4)}$$

$$= \frac{2x}{6(1-x^4)} = \frac{x}{3(1-x^4)}$$

EJERCICIO 11

$$1. \frac{1}{m-n} + \frac{m}{n^2-m^2} = \frac{1}{m-n} - \frac{m}{m^2-n^2} \quad mcm=m^2-n^2$$

$$= \frac{m+n-m}{m^2-n^2} = \frac{n}{m^2-n^2}$$

$$2. \frac{x^2}{x^2-xy} - \frac{2x}{y-x} = \frac{x^2}{x(x-y)} + \frac{2x}{x-y} = \frac{x}{x-y} + \frac{2x}{x-y}$$

$$mcm=x-y$$

$$= \frac{x+2x}{x-y} = \frac{3x}{x-y}$$

$$3. \frac{1}{2x-x^2} + \frac{x}{x^2-4} = \frac{1}{x(2-x)} - \frac{x}{4-x^2}$$

$$mcm=x(4-x^2)$$

$$= \frac{2+x-x^2}{x(4-x^2)} = \frac{(2-x)(1+x)}{x(2-x)(x+2)} = \frac{1+x}{x(2+x)}$$

$$4. \frac{a+b}{a^2-ab} + \frac{a}{b^2-a^2} = \frac{a+b}{a(a-b)} - \frac{a}{a^2-b^2}$$

$$mcm=a(a^2-b^2)$$

$$= \frac{(a+b)(a+b) - a^2}{a(a^2-b^2)} = \frac{a^2+2ab+b^2-a^2}{a(a^2-b^2)} = \frac{2ab+b^2}{a(a^2-b^2)}$$

$$5. \frac{x-4}{x^2-2x-3} - \frac{x}{6-2x} = \frac{x-4}{(x-3)(x+1)} + \frac{x}{2(x-3)}$$

$$mcm=2(x-3)(x+1)$$

$$= \frac{2(x-4) + x(x+1)}{2(x-3)(x+1)} = \frac{2x-8+x^2+x}{2(x-3)(x+1)} = \frac{x^2+3x-8}{2(x-3)(x+1)}$$

$$6. \frac{1}{x^2+2x-8} + \frac{1}{(2-x)(x+3)} - \frac{1}{(x+4)(x-2)} - \frac{1}{(x-2)(x+3)}$$

$$mcm=(x-2)(x+3)(x+4)$$

$$= \frac{x+3-(x+4)}{(x-2)(x+3)(x+4)}$$

$$= \frac{x+3-x-4}{(x-2)(x+3)(x+4)}$$

$$= \frac{-1}{(x-2)(x+3)(x+4)} = \frac{1}{(2-x)(x+3)(x+4)}$$

$$7. \frac{1}{2x+2} + \frac{2}{1-x} + \frac{7}{4x-4} = \frac{1}{2(x+1)} - \frac{2}{x-1} + \frac{7}{4(x-1)}$$

$$mcm=4(x^2-1)$$

$$= \frac{2(x-1) - 8(x+1) + 7(x+1)}{4(x^2-1)}$$

$$= \frac{2x-2-8x-8+7x+7}{4(x^2-1)} = \frac{x-3}{4(x^2-1)}$$

$$8. \frac{2a}{a+3} + \frac{3a}{a-3} + \frac{2a}{9-a^2} = \frac{2a}{a+3} + \frac{3a}{a-3} - \frac{2a}{a^2-9}$$

$$mcm = a^2 - 9$$

$$= \frac{2a(a-3) + 3a(a+3) - 2a}{a^2 - 9}$$

$$= \frac{2a^2 - 6a + 3a^2 + 9a - 2a}{a^2 - 9} = \frac{5a^2 + a}{a^2 - 9}$$

$$9. \frac{x+3y}{y+x} + \frac{3y^2}{x^2-y^2} - \frac{x}{y-x} = \frac{x+3y}{y+x} + \frac{3y^2}{x^2-y^2} - \frac{x}{x-y}$$

$$mcm = x^2 - y^2$$

$$= \frac{(x+3y)(x-y) + 3y^2 + x(x+y)}{x^2 - y^2}$$

$$= \frac{x^2 - xy + 3xy - 3y^2 + 3y^2 + x^2 + xy}{x^2 - y^2} = \frac{2x^2 + 3xy}{x^2 - y^2}$$

$$10. \frac{x}{x^2+2x-3} + \frac{x-3}{(1-x)(x+2)} + \frac{1}{x+2}$$

$$= \frac{x}{(x+3)(x-1)} - \frac{x-3}{(x-1)(x+2)} + \frac{1}{x+2}$$

$$mcm = (x+3)(x+2)(x-1)$$

$$= \frac{x(x+2) - (x-3)(x+3) + (x+3)(x-1)}{(x+3)(x+2)(x-1)}$$

$$= \frac{x^2 + 2x - x^2 + 9 + x^2 + 2x - 3}{(x+3)(x+2)(x-1)} = \frac{x^2 + 4x + 6}{(x+3)(x+2)(x-1)}$$

$$11. \frac{3}{2a+2} - \frac{1}{4a-4} - \frac{4}{8-8a^2} = \frac{3}{2(a+1)} - \frac{1}{4(a-1)} + \frac{4}{8(a^2-1)}$$

$$mcm = 8(a^2 - 1)$$

$$= \frac{12(a-1) - 2(a+1) + 4}{8(a^2 - 1)}$$

$$= \frac{12a - 12 - 2a - 2 + 4}{8(a^2 - 1)} = \frac{10a - 10}{8(a^2 - 1)} = \frac{10(a-1)}{8(a-1)(a+1)} = \frac{5}{4(a+1)}$$

$$12. \frac{1}{a-3} + \frac{a+1}{(3-a)(a-2)} + \frac{2}{(2-a)(1-a)}$$

$$= \frac{1}{a-3} - \frac{a+1}{(a-3)(a-2)} + \frac{2}{(a-2)(a-1)}$$

$$mcm = (a-3)(a-2)(a-1)$$

$$= \frac{(a-2)(a-1) - (a+1)(a-1) + 2(a-3)}{(a-3)(a-2)(a-1)}$$

$$= \frac{a^2 - 3a + 2 - a^2 + 1 + 2a - 6}{(a-3)(a-2)(a-1)}$$

$$= \frac{-a-3}{(a-3)(a-2)(a-1)} = \frac{a+3}{(3-a)(a-2)(a-1)}$$

$$13. \frac{2x}{x-1} + \frac{2x^3+2x^2}{1-x^3} + \frac{1}{x^2+x+1}$$

$$= \frac{2x}{x-1} - \frac{2x^2(x+1)}{x^3-1} + \frac{1}{x^2+x+1}$$

$$mcm = x^3 - 1$$

$$= \frac{2x(x^2+x+1) - 2x^2(x+1) + (x-1)}{x^3 - 1}$$

$$= \frac{2x^3 + 2x^2 + 2x - 2x^3 - 2x^2 + x - 1}{x^3 - 1} = \frac{3x - 1}{x^3 - 1}$$

$$\begin{aligned}
 14. \quad & \frac{x+2}{3x-1} + \frac{x+1}{3-2x} + \frac{4x^2+6x+3}{6x^2-11x+3} \\
 &= \frac{x+2}{3x-1} - \frac{x+1}{2x-3} + \frac{4x^2+6x+3}{(2x-3)(3x-1)} \\
 & \text{mcm} = (2x-3)(3x-1) \\
 &= \frac{(x+2)(2x-3) - (x+1)(3x-1) + 4x^2+6x+3}{(2x-3)(3x-1)} \\
 &= \frac{2x^2-3x+4x-6-3x^2+x-3x+1+4x^2+6x+3}{(2x-3)(3x-1)} \\
 &= \frac{3x^2+5x-2}{(2x-3)(3x-1)} = \frac{(x+2)(3x-1)}{(2x-3)(3x-1)} = \frac{x+2}{2x-3}
 \end{aligned}$$

Multiplicación y división de expresiones algebraicas

EJERCICIO 12

$$1. \quad \frac{2a^2}{3b} \cdot \frac{6b^2}{4a} = \frac{12a^2b^2}{12ab} = ab$$

$$2. \quad \frac{x^2y}{5} \cdot \frac{10a^3}{3m^2} \cdot \frac{9m}{x^3} = \frac{90x^2ya^3m}{15x^3m^2} = \frac{6a^3y}{xm}$$

$$3. \quad \frac{5x^2}{7y^3} \cdot \frac{4y^2}{7m^3} \cdot \frac{14m}{5x^4} = \frac{14 \cdot 5 \cdot 4x^2my^2}{49 \cdot 5x^4m^3y^3} = \frac{8}{7x^2m^2y}$$

$$4. \quad \frac{5}{a} \cdot \frac{2a}{b^2} \cdot \frac{3b}{10} = \frac{10 \cdot 3ab}{10ab^2} = \frac{3}{b}$$

$$5. \quad \frac{2x^3}{15a^3} \cdot \frac{3a^2}{y} \cdot \frac{5x^2}{7xy^2} = \frac{15 \cdot 2a^2x^5}{15 \cdot 7a^3xy^3} = \frac{2x^4}{7ay^3}$$

$$6. \quad \frac{7a}{6m^2} \cdot \frac{3m}{10n^2} \cdot \frac{5n^4}{14ax} = \frac{7 \cdot 3 \cdot 5amn^4}{14 \cdot 6 \cdot 10am^2n^2x} = \frac{n^2}{8mx}$$

$$7. \quad \frac{2x^2+x}{6} + \frac{8}{4x+2} = \frac{x(2x+1)}{3} \cdot \frac{4}{2(2x+1)} = \frac{2x}{3}$$

$$8. \quad \frac{5x+25}{14} \cdot \frac{7x+7}{10x+50} = \frac{5(x+5)}{14} \cdot \frac{7(x+1)}{10(x+5)} = \frac{x+1}{4}$$

$$9. \frac{m+n}{mn-n^2} \cdot \frac{n^2}{m^2-n^2} = \frac{m+n}{n(m-n)} \cdot \frac{n^2}{(m+n)(m-n)} = \frac{n}{(m-n)^2} = \frac{n}{m^2-2mn+n^2}$$

$$10. \frac{xy-2y^2}{x^2+xy} \cdot \frac{x^2+2xy+y^2}{x^2-2xy} = \frac{y(x-2y)}{x(x+y)} \cdot \frac{(x+y)^2}{x(x-2y)} = \frac{y(x+y)}{x^2} = \frac{xy+y^2}{x^2}$$

$$11. \frac{x^2-4xy+4y^2}{x^2+2xy} \cdot \frac{x^2}{x^2-4y^2} = \frac{(x-2y)^2}{x(x+2y)} \cdot \frac{x^2}{(x+2y)(x-2y)} = \frac{x(x-2y)}{(x+2y)^2} = \frac{x^2-2xy}{x^2+4xy+y^2}$$

$$12. \frac{2x^2+2x}{2x^2} \cdot \frac{x^2-3x}{x^2-2x-3} = \frac{2x(x+1)}{2x^2} \cdot \frac{x(x-3)}{(x-3)(x+1)} = 1$$

$$13. \frac{a^2-ab+a-b}{a^2+2a+1} \cdot \frac{3}{6a^2-6ab} = \frac{(a+1)(a-b)}{(a+1)(a+1)} \cdot \frac{3}{6a(a-b)} = \frac{1}{2a(a+1)} = \frac{1}{2a^2+2a}$$

$$14. \frac{(x-y)^3}{x^3-1} \cdot \frac{x^2+x+1}{(x-y)^2} = \frac{(x-y)}{(x-1)(x^2+x+1)} \cdot \frac{x^2+x+1}{x-1} = \frac{x-y}{x-1}$$

$$15. \frac{2a-2}{2a^2-50} \cdot \frac{a^2-4a-5}{3a+3} = \frac{2(a-1)}{2(a^2-25)} \cdot \frac{(a-5)(a+1)}{3(a+1)} = \frac{(a-1)(a-5)}{3(a+5)(a-5)} = \frac{a-1}{3(a+5)} = \frac{a-1}{3a+15}$$

$$16. \frac{2x^2-3x-2}{6x+3} \cdot \frac{3x+6}{x^2-4} = \frac{(x-2)(2x+1)}{3(2x+1)} \cdot \frac{3(x+2)}{(x-2)(x+2)} = 1$$

$$17. \frac{y^2+9y+18}{y-5} \cdot \frac{5y-25}{5y+15} = \frac{(y+6)(y+3)}{y-5} \cdot \frac{5(y-5)}{5(y+3)} = y+6$$

$$18. \frac{x^3+2x^2-3x}{4x^2+8x+3} \cdot \frac{2x^2+3x}{x^2-x} = \frac{x(x^2+2x-3)}{(2x+3)(2x+1)} \cdot \frac{x(2x+3)}{x(x-1)} = \frac{x(x+3)(x-1)}{(2x+1)(x-1)} = \frac{x(x+3)}{2x+1} = \frac{x^2+3x}{2x+1}$$

$$19. \frac{x^3-27}{a^3-1} \cdot \frac{a^2+a+1}{x^2+3x+9} = \frac{(x-3)(x^2+3x+9)}{(a-1)(a^2+a+1)} \cdot \frac{a^2+a+1}{x^2+3x+9} = \frac{x-3}{a-1}$$

$$21. \frac{1-x}{a+1} \cdot \frac{a^2+a}{x-x^2} \cdot \frac{x^2}{a} = \frac{1-x}{a+1} \cdot \frac{a(a+1)}{x(1-x)} \cdot \frac{x^2}{a} = x$$

$$22. \frac{x^2+2x}{x^2-16} \cdot \frac{x^2-2x-8}{x^3+x^2} \cdot \frac{x^2+4x}{x^2+4x+4} = \frac{x(x+2)}{(x-4)(x+4)} \cdot \frac{(x-4)(x+2)}{x^2(x+1)} \cdot \frac{x(x+4)}{(x+2)(x+4)} = \frac{x^2(x+2)^2(x-4)(x+4)}{x^2(x+2)^2(x-4)(x+4)(x+1)} = \frac{1}{x+1}$$

$$23. \frac{(m+n)^2-x^2}{(m+x)^2-n^2} \cdot \frac{(m-n)^2-x^2}{m^2+mn-mx} = \frac{(m+n+x)(m+n-x)}{(m+n+x)(m-n+x)} \cdot \frac{(m-n+x)(m-n-x)}{m(m+n-x)} = \frac{m-n-x}{m}$$

$$24. \frac{2a^3+2ab^2}{2ax^2-2ax} \cdot \frac{x^3-x}{a^2x+b^2x} \cdot \frac{x}{x+1} = \frac{2a(a^2+b^2)}{2ax(x-1)} \cdot \frac{x(x^2-1)}{x(a^2+b^2)} \cdot \frac{x}{x+1} = 1$$

$$25. \frac{a^2-5a+6}{3a-15} \cdot \frac{6a}{a^2-a-30} \cdot \frac{a^2-25}{2a-4} = \frac{(a-3)(a-2)}{3(a-5)} \cdot \frac{6a}{(a-6)(a+5)} \cdot \frac{(a+5)(a-5)}{2(a-2)} = \frac{a(a-3)}{a-6} = \frac{a^2-3a}{a-6}$$

$$26. \frac{x^2-3xy-10y^2}{x^2-2xy-8y^2} \cdot \frac{x^2-16y^2}{x^2+4xy} \cdot \frac{x^2-6xy}{x+2y} = \frac{(x-5y)(x+2y)}{(x-4y)(x+2y)} \cdot \frac{(x+4y)(x-4y)}{x(x+4y)} \cdot \frac{x(x-6y)}{x+2y} = \frac{(x-5y)(x-6y)}{x+2y} = \frac{x^2-11xy+30y^2}{x+2y}$$

$$27. \frac{x^2+4ax+4a^2}{3ax-6a^2} \cdot \frac{2ax-4a^2}{ax+a} \cdot \frac{6a+6x}{x^2+3ax+2a^2} = \frac{(x+2a)^2}{3a(x-2a)} \cdot \frac{2a(x-2a)}{a(x+1)} \cdot \frac{6(a+x)}{(x+2a)(x+a)} = \frac{4(x+2a)}{a(x+1)} = \frac{4x+8a}{ax+a}$$

$$28. \frac{a^2-81}{2a^2+10a} \cdot \frac{a+11}{a^2-36} \cdot \frac{2a-12}{2a+18} \cdot \frac{a^3+5a^2}{2a+22} = \frac{(a+9)(a-9)}{2a(a+5)} \cdot \frac{a+11}{(a+6)(a-6)} \cdot \frac{2(a-6)}{2(a+9)} \cdot \frac{a^2(a+5)}{2(a+11)}$$

$$\begin{aligned}
 29. \frac{a^2+7a+10}{a^2-6a-7} \cdot \frac{a^2-3a-4}{a^2+2a-15} \cdot \frac{a^3-2a^2-3a}{a^2-2a-8} \\
 &= \frac{(a+5)(a+2)}{(a-7)(a+1)} \cdot \frac{(a-4)(a+1)}{(a+5)(a-3)} \cdot \frac{a(a^2-2a-3)}{(a-4)(a+2)} \\
 &= \frac{a(a-3)(a+1)}{(a-3)(a-7)} = \frac{a(a+1)}{a-7} = \frac{a^2+a}{a-7}
 \end{aligned}$$

$$\begin{aligned}
 30. \frac{x^4+27x}{x^3-x^2+x} \cdot \frac{x^4+x}{x^4-3x^3+9x^2} \cdot \frac{1}{x(x+3)^2} \cdot \frac{x^2}{x-3} \\
 &= \frac{x(x^3+27)}{x(x^2-x+1)} \cdot \frac{x(x^3+1)}{x^2(x^2-3x+9)} \cdot \frac{x}{(x+3)^2(x-3)} \\
 &= \frac{(x+3)(x^2-3x+9)}{x^2-x+1} \cdot \frac{(x+1)(x^2-x+1)}{x^2-3x+9} \cdot \frac{1}{(x^2-9)(x+3)} \\
 &= \frac{x+1}{x^2-9}
 \end{aligned}$$

EJERCICIO 13

$$\begin{aligned}
 1. \left(a + \frac{a}{b}\right) \left(a - \frac{a}{b+1}\right) \\
 \Rightarrow a + \frac{a}{b} = \frac{ab+a}{b} = \frac{a(b+1)}{b} \\
 \Rightarrow a - \frac{a}{b+1} = \frac{a(b+1)-a}{b+1} = \frac{ab+a-a}{b+1} = \frac{ab}{b+1} \\
 \Rightarrow \frac{a(b+1)}{b} \cdot \frac{ab}{b+1} = a^2
 \end{aligned}$$

$$\begin{aligned}
 2. \left(x - \frac{2}{x+1}\right) \left(x + \frac{1}{x+2}\right) \\
 \Rightarrow x - \frac{2}{x+1} = \frac{x(x+1)-2}{x+1} = \frac{x^2+x-2}{x+1} = \frac{(x+2)(x-1)}{x+1} \\
 \Rightarrow x + \frac{1}{x+2} = \frac{x(x+2)+1}{x+2} = \frac{x^2+2x+1}{x+2} = \frac{(x+1)(x+1)}{x+2} \\
 \Rightarrow \frac{(x+2)(x-1)}{x+1} \cdot \frac{(x+1)^2}{x+2} = (x-1)(x+1) = x^2-1
 \end{aligned}$$

$$\begin{aligned}
 3. \left(1 - \frac{x}{a+x}\right) \left(1 + \frac{x}{a}\right) \\
 \Rightarrow 1 - \frac{x}{a+x} = \frac{a+x-x}{a+x} = \frac{a}{a+x} \\
 \Rightarrow 1 + \frac{x}{a} = \frac{a+x}{a} \\
 \Rightarrow \frac{a}{a+x} \cdot \frac{a+x}{a} = 1
 \end{aligned}$$

$$\begin{aligned}
 4. \left(a + \frac{ab}{a-b}\right) \left(1 - \frac{b^2}{a^2}\right) \\
 \Rightarrow a + \frac{ab}{a-b} = \frac{a(a-b)+ab}{a-b} = \frac{a^2-ab+ab}{a-b} = \frac{a^2}{a-b} \\
 \Rightarrow 1 - \frac{b^2}{a^2} = \frac{a^2-b^2}{a^2} \\
 \Rightarrow \frac{a^2}{a-b} \cdot \frac{a^2-b^2}{a^2} = \frac{(a+b)(a-b)}{(a-b)} = a+b
 \end{aligned}$$

$$\begin{aligned}
 5. \left(x+2 - \frac{12}{x+1}\right) \left(x-2 + \frac{10-3x}{x+5}\right) \\
 \Rightarrow x+2 - \frac{12}{x+1} = \frac{(x+2)(x+1)-12}{x+1} \\
 = \frac{x^2+3x+2-12}{x+1} = \frac{x^2+3x-10}{x+1} = \frac{(x+5)(x-2)}{x+1} \\
 \Rightarrow x-2 + \frac{10-3x}{x+5} = \frac{(x-2)(x+5)+10-3x}{x+5} \\
 = \frac{(x-2)(x+5)+10-3x}{x+5} = \frac{x^2+3x-10+10-3x}{x+5} = \frac{x^2}{x+5}
 \end{aligned}$$

$$\Rightarrow \frac{(x+5)(x-2)}{x+1} \cdot \frac{x^2}{x+5} = \frac{x^2(x-2)}{x+1} = \frac{x^3-2x^2}{x+1}$$

$$\begin{aligned}
 6. \left(1 + \frac{x}{y}\right) \left(x - \frac{x^2}{x+y}\right) \\
 \Rightarrow 1 + \frac{x}{y} = \frac{y+x}{y} \\
 \Rightarrow x - \frac{x^2}{x+y} = \frac{x(x+y)-x^2}{x+y} = \frac{x^2+xy-x^2}{x+y} = \frac{xy}{x+y} \\
 \Rightarrow \frac{y+x}{y} \cdot \frac{xy}{x+y} = x
 \end{aligned}$$

$$\begin{aligned}
 7. \left(a+x - \frac{ax+x^2}{a+2x}\right) \left(1 + \frac{x}{a+x}\right) \\
 \Rightarrow a+x - \frac{ax+x^2}{a+2x} = \frac{(a+x)(a+2x)-(ax+x^2)}{a+2x} \\
 = \frac{a^2+3ax+2x^2-ax-x^2}{a+2x} = \frac{a^2+2ax+x^2}{a+2x} = \frac{(a+x)^2}{a+2x} \\
 \Rightarrow 1 + \frac{x}{a+x} = \frac{a+x+x}{a+x} = \frac{a+2x}{a+x} \\
 \Rightarrow \frac{(a+x)^2}{a+2x} \cdot \frac{a+2x}{a+x} = a+x
 \end{aligned}$$

$$\begin{aligned}
 8. \quad & \left(x - \frac{x^3 - 6x}{x^2 - 25}\right) \left(x + 1 - \frac{8}{x+3}\right) \\
 & \Rightarrow x - \frac{x^3 - 6x}{x^2 - 25} = \frac{x(x^2 - 25) - (x^3 - 6x)}{x^2 - 25} \\
 & = \frac{x^3 - 25x - x^3 + 6x}{x^2 - 25} = \frac{-19x}{x^2 - 25} = \frac{19x}{25 - x^2} \\
 & \Rightarrow x + 1 - \frac{8}{x+3} = \frac{(x+1)(x+3) - 8}{x+3} \\
 & = \frac{x^2 + 4x + 3 - 8}{x+3} = \frac{x^2 + 4x - 5}{x+3} = \frac{(x+5)(x-1)}{x+3} \\
 & \Rightarrow \frac{19x}{25 - x^2} \cdot \frac{(x+5)(x-1)}{x+3} \\
 & = \frac{19x(x-1)}{(5-x)(x+3)} = \frac{19x^2 - 19x}{5x + 15 - x^2 - 3x} = \frac{19x^2 - 19x}{15 + 2x - x^2}
 \end{aligned}$$

$$\begin{aligned}
 10. \quad & \left(a + 2x - \frac{14x^2}{2a+x}\right) \left(a - x + \frac{a^2 + 5x^2}{a+4x}\right) \\
 & \Rightarrow a + 2x - \frac{14x^2}{2a+x} \\
 & = \frac{(a+2x)(2a+x) - 14x^2}{2a+x} \\
 & = \frac{2a^2 + 5ax + 2x^2 - 14x^2}{2a+x} = \frac{2a^2 + 5ax - 12x^2}{2a+x} = \frac{(a+4x)(2a-3x)}{2a+x} \\
 & \Rightarrow a - x + \frac{a^2 + 5x^2}{a+4x} \\
 & = \frac{(a-x)(a+4x) + a^2 + 5x^2}{a+4x} \\
 & = \frac{a^2 + 3ax - 4x^2 + a^2 + 5x^2}{a+4x} = \frac{2a^2 + 3ax + x^2}{a+4x} = \frac{(a+x)(2a+x)}{a+4x} \\
 & \Rightarrow \frac{(a+4x)(2a-3x)}{2a+x} \cdot \frac{(a+x)(2a+x)}{a+4x} \\
 & = (2a-3x)(a+x) = 2a^2 - ax - 3x^2
 \end{aligned}$$

$$\begin{aligned}
 9. \quad & \left(m - \frac{mn}{m+n}\right) \left(1 + \frac{n^3}{m^3}\right) \\
 & \Rightarrow m - \frac{mn}{m+n} = \frac{m(m+n) - mn}{m+n} = \frac{m^2 + mn - mn}{m+n} = \frac{m^2}{m+n} \\
 & \Rightarrow 1 + \frac{n^3}{m^3} = \frac{m^3 + n^3}{m^3} = \frac{(m+n)(m^2 - mn + n^2)}{m^3} \\
 & \Rightarrow \frac{m^2}{m+n} \cdot \frac{(m+n)(m^2 - mn + n^2)}{m^3} = \frac{m^2 - mn + n^2}{m}
 \end{aligned}$$

$$\begin{aligned}
 11. \quad & \left(1 + \frac{a}{b}\right) \left(1 - \frac{b}{a}\right) \left(1 + \frac{b^2}{a^2 - b^2}\right) \\
 & \Rightarrow 1 + \frac{a}{b} = \frac{b+a}{b} \\
 & \Rightarrow 1 - \frac{b}{a} = \frac{a-b}{a} \\
 & \Rightarrow 1 + \frac{b^2}{a^2 - b^2} = \frac{a^2}{a^2 - b^2} \\
 & \Rightarrow \frac{b+a}{b} \cdot \frac{a-b}{a} \cdot \frac{a^2}{(a+b)(a-b)} = \frac{a^2}{ab} = \frac{a}{b}
 \end{aligned}$$

$$\begin{aligned}
 12. \quad & \left(2 + \frac{2}{x+1}\right) \left(3 - \frac{6}{x+2}\right) \left(1 + \frac{1}{x}\right) \\
 & \Rightarrow 2 + \frac{2}{x+1} = \frac{2(x+1) + 2}{x+1} = \frac{2x+4}{x+1} = \frac{2(x+2)}{x+1} \\
 & \Rightarrow 3 - \frac{6}{x+2} = \frac{3(x+2) - 6}{x+2} = \frac{3x+6-6}{x+2} = \frac{3x}{x+2} \\
 & \Rightarrow 1 + \frac{1}{x} = \frac{x+1}{x} \\
 & \Rightarrow \frac{2(x+2)}{x+1} \cdot \frac{3x}{x+2} \cdot \frac{x+1}{x} = 6
 \end{aligned}$$

EJERCICIO 14

$$1. \frac{x^2}{3y^2} \div \frac{2x}{y^3} = \frac{x^2}{3y^2} \cdot \frac{y^3}{2x} = \frac{xy}{6}$$

$$2. \frac{3a^2b}{5x^2} \div a^2b^3 = \frac{3a^2b}{5x^2} \cdot \frac{1}{a^2b^3} = \frac{3}{5b^2x^2}$$

$$3. \frac{5m^2}{7n^3} \div \frac{10m^4}{14an^4} = \frac{5m^2}{7n^3} \cdot \frac{14an^4}{10m^4} = \frac{an}{m^2}$$

$$4. 6a^2x^3 \div \frac{a^2x}{5} = 6a^2x^3 \cdot \frac{5}{a^2x} = 30x^2$$

$$5. \frac{15m^2}{19ax^3} \div \frac{20y^2}{38a^3x^4} = \frac{15m^2}{19ax^3} \cdot \frac{38a^3x^4}{20y^2} = \frac{3m^2a^2x}{2y^2}$$

$$6. \frac{11x^2y^3}{7m^2} \div 22y^4 = \frac{11x^2y^3}{7m^2} \cdot \frac{1}{22y^4} = \frac{x^2}{14m^2y}$$

$$7. \frac{x-1}{3} \div \frac{2x-2}{6} = \frac{x-1}{3} \cdot \frac{6}{2(x-1)} = 1$$

$$8. \frac{3a^2}{a^2 + 6ab + 9b^2} \div \frac{5a^3}{a^2b + 3ab^2} = \frac{3a^2}{(a+3b)^2} \cdot \frac{ab(a+3b)}{5a^3} = \frac{3b}{5(a+3b)}$$

$$9. \frac{x^3 - x}{2x^2 + 6x} \div \frac{5x^2 - 5x}{2x + 6} = \frac{x(x^2 - 1)}{2x(x+3)} \cdot \frac{2(x+3)}{5x(x-1)} = \frac{x+1}{5x}$$

$$10. \frac{1}{a^2 - a - 30} \div \frac{2}{a^2 + a - 42} = \frac{1}{(a-6)(a+5)} \cdot \frac{(a+7)(a-6)}{2} = \frac{a+7}{2(a+5)}$$

$$11. \frac{20x^2 - 30x}{15x^3 + 15x^2} \div \frac{4x - 6}{x+1} = \frac{5x(4x-6)}{15x^2(x+1)} \cdot \frac{x+1}{4x-6} = \frac{1}{3x}$$

$$\begin{aligned}
 12. \quad & \frac{a^2 - 6a + 5}{a^2 - 15a + 56} \div \frac{a^2 + 2a - 35}{a^2 - 5a - 24} \\
 &= \frac{(a-5)(a-1)}{(a-8)(a-7)} \cdot \frac{(a-8)(a+3)}{(a+7)(a-5)} \\
 &= \frac{(a-1)(a+3)}{(a-7)(a+7)} = \frac{a^2 + 2a - 3}{a^2 - 49} \\
 13. \quad & \frac{8x^2 + 26x + 15}{16x^2 - 9} \div \frac{6x^2 + 13x - 5}{9x^2 - 1} \\
 &= \frac{(2x+5)(4x+3)}{(4x+3)(4x-3)} \cdot \frac{(3x+1)(3x-1)}{(2x+5)(3x-1)} = \frac{3x+1}{4x-3} \\
 14. \quad & \frac{x^3 - 121x}{x^2 - 49} \div \frac{x^2 - 11x}{x+7} \\
 &= \frac{x(x^2 - 121)}{(x+7)(x-7)} \cdot \frac{(x+7)}{x(x-11)} = \frac{x+11}{x-7} \\
 15. \quad & \frac{ax^2 + 5}{4a^2 - 1} \div \frac{a^3x^2 + 5a^2}{2a - 1} \\
 &= \frac{ax^2 + 5}{(2a+1)(2a-1)} \cdot \frac{2a-1}{a^2(ax^2 + 5)} = \frac{1}{a^2(2a+1)} \\
 16. \quad & \frac{a^4 - 1}{a^3 + a^2} \div \frac{a^4 + 4a^2 + 3}{3a^3 + 9a} \\
 &= \frac{(a^2+1)(a^2-1)}{a^2(a+1)} \cdot \frac{3a(a^2+3)}{(a^2+3)(a^2+1)} = \frac{3(a-1)}{a} \\
 17. \quad & \frac{x^3 + 125}{x^2 - 64} \div \frac{x^3 - 5x^2 + 25x}{x^2 + x - 56} \\
 &= \frac{(x+5)(x^2 - 5x + 25)}{(x+8)(x-8)} \cdot \frac{(x+8)(x-7)}{x(x^2 - 5x + 25)} \\
 &= \frac{(x+5)(x-7)}{x(x-8)} = \frac{x^2 - 2x - 35}{x^2 - 8x} \\
 18. \quad & \frac{16x^2 - 24xy + 9y^2}{16x - 12y} \div \frac{64x^3 - 27y^3}{32x^2 + 24xy + 18y^2} \\
 &= \frac{(4x-3y)^2}{4(4x-3y)} \cdot \frac{2(16x^2 + 12xy + 9y^2)}{(4x-3y)(16x^2 + 12xy + 9y^2)} = \frac{1}{2} \\
 19. \quad & \frac{a^2 - 6a}{a^3 + 3a^2} \div \frac{a^2 + 3a - 54}{a^2 + 9a} = \frac{a(a-6)}{a^2(a+3)} \cdot \frac{a(a+9)}{(a+9)(a-6)} = \frac{1}{a+3} \\
 20. \quad & \frac{15x^2 + 7x - 2}{25x^3 - x} \div \frac{6x^2 + 13x + 6}{25x^2 + 10x + 1} \\
 &= \frac{(3x+2)(5x-1)}{x(5x-1)(5x+1)} \cdot \frac{(5x+1)^2}{(2x+3)(3x+2)} = \frac{5x+1}{x(2x+3)} = \frac{5x+1}{2x^2 + 3x} \\
 21. \quad & \frac{x^3 - 1}{2x^2 - 2x + 2} \div \frac{7x^2 + 7x + 7}{7x^3 + 7} \\
 &= \frac{(x-1)(x^2 + x + 1)}{2(x^2 - x + 1)} \cdot \frac{7(x^3 + 1)}{7(x^2 + x + 1)} \\
 &= \frac{(x-1)}{2(x^2 - x + 1)} \cdot (x+1)(x^2 - x + 1) = \frac{(x-1)(x+1)}{2} = \frac{x^2 - 1}{2} \\
 22. \quad & \frac{2mx - 2my + nx - ny}{3x - 3y} \div 8m + 4n \\
 &= \frac{(2m+n)(x-y)}{3(x-y)} \cdot \frac{1}{4(2m+n)} = \frac{1}{12} \\
 23. \quad & \frac{x^2 - 6x + 9}{4x^2 - 1} \div \frac{x^2 + 5x - 24}{2x^2 + 17x + 8} \\
 &= \frac{(x-3)^2}{(2x+1)(2x-1)} \cdot \frac{(2x+1)(x+8)}{(x+8)(x-3)} = \frac{x-3}{2x-1} \\
 24. \quad & \frac{2a^2 + 7ab - 15b^2}{a^3 + 4a^2b} \div \frac{a^2 - 3ab - 40b^2}{a^2 - 4ab - 32b^2} \\
 &= \frac{(a+5b)(2a-3b)}{a^2(a+4b)} \cdot \frac{(a-8b)(a+4b)}{(a-8b)(a+5b)} = \frac{2a-3b}{a^2}
 \end{aligned}$$

EJERCICIO 15

$$\begin{aligned}
 1. \quad & \left(1 + \frac{a}{a+b}\right) \div \left(1 + \frac{2a}{b}\right) \\
 & \Rightarrow 1 + \frac{a}{a+b} = \frac{a+b+a}{a+b} = \frac{2a+b}{a+b} \\
 & \Rightarrow 1 + \frac{2a}{b} = \frac{b+2a}{b} \\
 & \Rightarrow \frac{2a+b}{a+b} \div \frac{b+2a}{b} = \frac{2a+b}{a+b} \cdot \frac{b}{b+2a} = \frac{b}{a+b} \\
 2. \quad & \left(x - \frac{2}{x+1}\right) \div \left(x - \frac{x}{x+1}\right) \\
 & \Rightarrow x - \frac{2}{x+1} = \frac{x(x+1) - 2}{x+1} = \frac{x^2 + x - 2}{x+1} \\
 & \Rightarrow x - \frac{x}{x+1} = \frac{x(x+1) - x}{x+1} = \frac{x^2}{x+1} \\
 & \Rightarrow \frac{x^2 + x - 2}{x+1} \div \frac{x^2}{x+1} = \frac{x^2 + x - 2}{x+1} \cdot \frac{x+1}{x^2} = \frac{x^2 + x - 2}{x^2}
 \end{aligned}$$

$$\begin{aligned}
 3. \left(1-a+\frac{a^2}{1+a}\right) \div \left(1+\frac{2}{a^2-1}\right) \\
 \Rightarrow 1-a+\frac{a^2}{1+a} = \frac{(1-a)(1+a)+a^2}{1+a} = \frac{1-a^2+a^2}{1+a} = \frac{1}{1+a} \\
 \Rightarrow 1+\frac{2}{a^2-1} = \frac{a^2-1+2}{a^2-1} = \frac{a^2+1}{a^2-1} \\
 \Rightarrow \frac{1}{1+a} \div \frac{a^2+1}{a^2-1} = \frac{1}{1+a} \cdot \frac{a^2-1}{a^2+1} = \frac{a-1}{a^2+1}
 \end{aligned}$$

$$\begin{aligned}
 4. \left(x+\frac{2}{x+3}\right) \div \left(x+\frac{3}{x+4}\right) \\
 \Rightarrow x+\frac{2}{x+3} = \frac{x(x+3)+2}{x+3} = \frac{x^2+3x+2}{x+3} = \frac{(x+1)(x+2)}{x+3} \\
 \Rightarrow x+\frac{3}{x+4} = \frac{x(x+4)+3}{x+4} = \frac{x^2+4x+3}{x+4} = \frac{(x+3)(x+1)}{x+4} \\
 \Rightarrow \frac{(x+1)(x+2)}{x+3} \div \frac{(x+3)(x+1)}{x+4} \\
 = \frac{(x+1)(x+2)}{x+3} \cdot \frac{x+4}{(x+3)(x+1)} = \frac{(x+2)(x+4)}{(x+3)^2} = \frac{x^2+6x+8}{x^2+6x+9}
 \end{aligned}$$

$$\begin{aligned}
 5. \left(a+b+\frac{b^2}{a-b}\right) \div \left(1-\frac{b}{a+b}\right) \\
 \Rightarrow a+b+\frac{b^2}{a-b} = \frac{(a+b)(a-b)+b^2}{a-b} = \frac{a^2-b^2+b^2}{a-b} = \frac{a^2}{a-b} \\
 \Rightarrow 1-\frac{b}{a+b} = \frac{a+b-b}{a+b} = \frac{a}{a+b} \\
 \Rightarrow \frac{a^2}{a-b} \div \frac{a}{a+b} = \frac{a^2}{a-b} \cdot \frac{a+b}{a} = \frac{a(a+b)}{a-b} = \frac{a^2+ab}{a-b}
 \end{aligned}$$

$$\begin{aligned}
 6. \left(1-\frac{1}{x^3+2}\right) \div \left(x+\frac{1}{x-1}\right) \\
 \Rightarrow 1-\frac{1}{x^3+2} = \frac{x^3+2-1}{x^3+2} = \frac{x^3+1}{x^3+2} = \frac{(x+1)(x^2-x+1)}{x^3+2} \\
 \Rightarrow x+\frac{1}{x-1} = \frac{x(x-1)+1}{x-1} = \frac{x^2-x+1}{x-1} \\
 \Rightarrow \frac{(x+1)(x^2-x+1)}{x^3+2} \div \frac{x^2-x+1}{x-1} \\
 = \frac{(x+1)(x^2-x+1)}{x^3+2} \cdot \frac{x-1}{x^2-x+1} = \frac{(x+1)(x-1)}{x^3-2} = \frac{x^2-1}{x^3+2}
 \end{aligned}$$

$$\begin{aligned}
 7. \left(x+\frac{1}{x+2}\right) \div \left(1+\frac{3}{x^2-4}\right) \\
 \Rightarrow x+\frac{1}{x+2} = \frac{x(x+2)+1}{x+2} = \frac{x^2+2x+1}{x+2} = \frac{(x+1)^2}{x+2} \\
 \Rightarrow 1+\frac{3}{x^2-4} = \frac{x^2-4+3}{x^2-4} = \frac{x^2-1}{x^2-4} = \frac{(x+1)(x-1)}{(x+2)(x-2)} \\
 \Rightarrow \frac{(x+1)^2}{x+2} \div \frac{(x+1)(x-1)}{(x+2)(x-2)} \\
 = \frac{(x+1)^2}{x+2} \cdot \frac{(x+2)(x-2)}{(x+1)(x-1)} = \frac{(x+1)(x-2)}{x-1} = \frac{x^2-x-2}{x-1}
 \end{aligned}$$

$$\begin{aligned}
 8. \left(n-\frac{2n-1}{n^2+2}\right) \div \left(n^2+1-\frac{n-1}{n}\right) \\
 \Rightarrow n-\frac{2n-1}{n^2+2} = \frac{n(n^2+2)-(2n-1)}{n^2+2} = \frac{n^3+2n-2n+1}{n^2+2} = \frac{n^3+1}{n^2+2} \\
 \Rightarrow n^2+1-\frac{n-1}{n} = \frac{n(n^2+1)-(n-1)}{n} = \frac{n^3+n-n+1}{n} = \frac{n^3+1}{n} \\
 \Rightarrow \frac{n^3+1}{n^2+2} \div \frac{n^3+1}{n} = \frac{n^3+1}{n^2+2} \cdot \frac{n}{n^3+1} = \frac{n}{n^2+2}
 \end{aligned}$$

EJERCICIO 16

$$1. \frac{3x}{4y} \cdot \frac{8y}{9x} \div \frac{z^2}{3x^2} = \frac{2}{3} \cdot \frac{3x^2}{z^2} = \frac{2x^2}{z^2}$$

$$2. \frac{5a}{b} \div \left(\frac{2a}{b^2} \cdot \frac{5x}{4a^2}\right) = \frac{5a}{b} \cdot \frac{2ab^2}{5x} = \frac{2a^2b}{x}$$

$$\begin{aligned}
 3. \frac{a+1}{a-1} \cdot \frac{3a-3}{2a+2} \div \frac{a^2+a}{a^2+a-2} \\
 = \frac{a+1}{a-1} \cdot \frac{3(a-1)}{2(a+1)} \cdot \frac{(a+2)(a-1)}{a(a+1)} \\
 = \frac{3(a+2)(a-1)}{2a(a+1)} = \frac{3a^2+3a-6}{2a^2+2a}
 \end{aligned}$$

$$\begin{aligned}
 4. \frac{64a^2-81b^2}{x^2-81} \cdot \frac{(x-9)^2}{8a-9b} \div \frac{8a^2+9ab}{(x+9)^2} \\
 = \frac{(8a+9b)(8a-9b)}{(x+9)(x-9)} \cdot \frac{(x-9)^2}{8a-9b} \cdot \frac{(x+9)^2}{a(8a+9b)} \\
 = \frac{(x-9)(x+9)}{a} = \frac{x^2-81}{a}
 \end{aligned}$$

$$\begin{aligned}
 5. \frac{x^2-x-12}{x^2-49} \cdot \frac{x^2-x-56}{x^2+x-20} \div \frac{x^2-5x-24}{x+5} \\
 = \frac{(x-4)(x+3)}{(x+7)(x-7)} \cdot \frac{(x-8)(x+7)}{(x+5)(x-4)} \cdot \frac{x+5}{(x-8)(x+3)} = \frac{1}{x-7}
 \end{aligned}$$

$$\begin{aligned}
 6. \frac{a^2-8a+7}{a^2-11a+30} \cdot \frac{a^2-36}{a^2-1} \div \frac{a^2-a-42}{a^2-4a-5} \\
 = \frac{(a-7)(a-1)}{(a-6)(a-5)} \cdot \frac{(a+6)(a-6)}{(a+1)(a-1)} \cdot \frac{(a-5)(a+1)}{(a-7)(a+6)} = 1
 \end{aligned}$$

$$7. \frac{x^4 - 27x}{x^2 + 7x - 30} \cdot \frac{x^2 + 20x + 100}{x^3 + 3x^2 + 9x} \div \frac{x^2 - 100}{x - 3} = \frac{x(x-3)(x^2 + 3x + 9)}{(x+10)(x-3)} \cdot \frac{(x+10)^2}{x(x^2 + 3x + 9)} \cdot \frac{x-3}{(x+10)(x-10)} = \frac{x-3}{x-10}$$

$$8. \frac{(a^2 + 1)}{3a - 6} \div \left(\frac{a^3 + a}{6a - 12} \cdot \frac{4x + 8}{x - 3} \right) = \frac{a^2 + 1}{3(a-2)} \div \frac{a(a^2 + 1) \cdot 4(x+2)}{6(a-2) \cdot x-3} = \frac{a^2 + 1}{3(a-2)} \cdot \frac{3(a-2)(x-3)}{2a(a^2 + 1)(x+2)} = \frac{x-3}{2a(x+2)} = \frac{x-3}{2ax+4a}$$

$$9. \frac{8x^2 - 10x - 3}{6x^2 + 13x + 6} \cdot \frac{4x^2 - 9}{3x^2 + 2x} \div \frac{8x^2 + 14x + 3}{9x^2 + 12x + 4} = \frac{(2x-3)(4x+1)}{(2x+3)(3x+2)} \cdot \frac{(2x+3)(2x-3)}{x(3x+2)} \cdot \frac{(3x+2)^2}{(2x+3)(4x+1)} = \frac{(2x-3)^2}{x(2x+3)} = \frac{4x^2 - 12x + 9}{2x^2 + 3x}$$

$$10. \frac{(a+b)^2 - c^2}{(a-b)^2 - c^2} \cdot \frac{(a+c)^2 - b^2}{a^2 + ab - ac} \div \frac{a+b+c}{a^2} = \frac{(a+b+c)(a+b-c)}{(a-b+c)(a-b-c)} \cdot \frac{(a+b+c)(a-b+c)}{a(a+b-c)} \cdot \frac{a^2}{a+b+c} = \frac{a(a+b+c)}{a-b-c} = \frac{a^2 + ab + ac}{a-b-c}$$

$$11. \frac{a^2 - 5a}{b + b^2} \div \left(\frac{a^2 + 6a - 55}{b^2 - 1} \cdot \frac{ax + 3a}{ab^2 + 11b^2} \right) = \frac{a(a-5)}{b(1+b)} \cdot \frac{b^2(b+1)(b-1)(a+11)}{a(a+11)(a-5)(x+3)} = \frac{b(b-1)}{x+3} = \frac{b^2 - b}{x+3}$$

$$12. \frac{m^3 + 6m^2n + 9mn^2}{2m^2n + 7mn^2 + 3n^3} \cdot \frac{4m^2 - n^2}{8m^2 - 2mn - n^2} \div \frac{m^3 + 27n^3}{16m^2 + 8mn + n^2}$$

$$= \frac{m(m+3n)^2}{n(m+3n)(2m+n)} \cdot \frac{(2m+n)(2m-n)}{(2m-n)(4m+n)} \cdot \frac{(4m+n)^2}{(m+3n)(m^2 - 3mn - 9n^2)} = \frac{m(4m+n)}{n(m^2 - 3mn + 9n^2)} = \frac{4m^2 + mn}{m^2n - 3mn^2 + 9n^3}$$

$$13. \frac{(a^2 - ax)^2}{a^2 + x^2} \cdot \frac{1}{a^3 + a^2x} \div \left(\frac{a^3 - a^2x}{a^2 + 2ax + x^2} \cdot \frac{a^2 - x^2}{a^3 + ax^2} \right)$$

$$= \frac{(a^2 - ax)^2}{a^2 + x^2} \cdot \frac{1}{a^2(a+x)} \cdot \frac{a(a+x)^2(a^2 + x^2)}{a(a^2 - ax)(a+x)(a-x)} = \frac{a^2 - ax}{a^2(a-x)} = \frac{a(a-x)}{a^2(a-x)} = \frac{1}{a}$$

$$14. \frac{(a^2 - 3a)^2}{9 - a^2} \cdot \frac{27 - a^3}{(a+3)^2 - 3a} \div \frac{a^4 - 9a^2}{(a^2 + 3a)^2}$$

$$= \frac{(a^2 - 3a)^2}{(3+a)(3-a)} \cdot \frac{(3-a)(9+3a+a^2)}{a^2 + 3a + 9} \cdot \frac{(a^2 + 3a)^2}{(a^2 - 3a)(a^2 + 3a)} = \frac{(a^2 - 3a)(a^2 + 3a)}{3+a} = \frac{a(a^2 - 3a)(a+3)}{a+3} = a^3 - 3a^2$$

EJERCICIO 137

$$1. \frac{a - \frac{a}{b}}{b - \frac{1}{b}} = \frac{\frac{ab - a}{b}}{\frac{b^2 - 1}{b}} = \frac{a(b-1)}{(b+1)(b-1)} = \frac{a}{b+1}$$

$$2. \frac{x^2 - \frac{1}{x}}{1 - \frac{1}{x}} = \frac{\frac{x^3 - 1}{x}}{\frac{x-1}{x}} = \frac{(x-1)(x^2 + x + 1)}{x-1} = x^2 + x + 1$$

$$3. \frac{\frac{a}{b} - \frac{b}{a}}{1 + \frac{b}{a}} = \frac{\frac{a^2 - b^2}{ab}}{\frac{a+b}{a}} = \frac{(a+b)(a-b)}{b(a+b)} = \frac{a-b}{b}$$

$$4. \frac{\frac{1}{m} + \frac{1}{n}}{\frac{1}{m} - \frac{1}{n}} = \frac{\frac{m+n}{mn}}{\frac{n-m}{mn}} = \frac{m+n}{n-m}$$

$$5. \frac{x + \frac{x}{2}}{x - \frac{x}{4}} = \frac{\frac{2x+x}{2}}{\frac{4x-x}{4}} = \frac{2(3x)}{3x} = 2$$

$$6. \frac{\frac{x}{y} - \frac{y}{x}}{1 + \frac{y}{x}} = \frac{\frac{x^2 - y^2}{xy}}{\frac{x+y}{x}} = \frac{(x+y)(x-y)}{y(x+y)} = \frac{x-y}{y}$$

$$7. \frac{x+4 + \frac{3}{x}}{x-4 - \frac{5}{x}} = \frac{\frac{x^2 + 4x + 3}{x}}{\frac{x^2 - 4x - 5}{x}} = \frac{(x+1)(x+3)}{(x-5)(x+1)} = \frac{x+3}{x-5}$$

$$8. \frac{a-4 + \frac{4}{a}}{1 - \frac{2}{a}} = \frac{\frac{a^2 - 4a + 4}{a}}{\frac{a-2}{a}} = \frac{(a-2)(a-2)}{(a-2)} = a-2$$

$$\begin{aligned}
 & \frac{2a^2 - b^2 - b}{\frac{a}{4a^2 + b^2} + 1} \\
 &= \frac{2a^2 - b^2 - ab}{\frac{a}{4a^2 + b^2 + 4ab}} \\
 &= \frac{4b(2a^2 - b^2 - ab)}{4a} \\
 &= \frac{4b(a-b)(2a+b)}{(2a+b)^2} \\
 &= \frac{4b(a-b)}{2a+b} = \frac{4ab - 4b^2}{2a+b} \\
 &= \frac{2 + \frac{3a}{5b}}{\frac{10b}{3} + \frac{10b+3a}{3}} = \frac{3}{5b} \\
 & 11. \frac{a-x + \frac{x^2}{a+x}}{a^2 - \frac{a^2}{a+x}} \\
 &= \frac{a^2 - x^2 + x^2}{\frac{a^2 - a^2}{a+x}} \\
 &= \frac{a^2 - x^2 + x^2}{\frac{a^3 + a^2x - a^2}{a+x}} \\
 &= \frac{a^2}{a^2(a+x-1)} = \frac{1}{a+x-1} \\
 & 12. \frac{a+5 - \frac{14}{8} \frac{a}{7}}{1 + \frac{a}{a^2} + \frac{a^2}{a^2}} \\
 &= \frac{a^2 + 5a - 14}{\frac{a^2 + 8a + 7}{a^2}} \\
 &= \frac{a(a+7)(a-2)}{(a+7)(a+1)} \\
 &= \frac{a(a-2)}{a+1} = \frac{a^2 - 2a}{a+1} \\
 & 13. \frac{\frac{1}{a} - \frac{9}{a^2} + \frac{20}{a^3}}{\frac{16}{a} - a} \\
 &= \frac{\frac{a^2 - 9a + 20}{a^3}}{\frac{16 - a^2}{a}} \\
 &= \frac{(a-5)(a-4)}{a^2(4+a)(4-a)} \\
 &= -\frac{a-5}{a^2(a+4)} = -\frac{a-5}{a^3+4a^2} = \frac{5-a}{a^3+4a^2}
 \end{aligned}$$

$$\begin{aligned}
 & 14. \frac{20x^2 + 7x - 6}{\frac{x}{\frac{4}{x^2} - 25}} \\
 &= \frac{(4x+3)(5x-2)}{\frac{x}{4-25x^2}} \\
 &= \frac{x(4x+3)(5x-2)}{(2+5x)(2-5x)} \\
 &= -\frac{4x^2 + 3x}{5x+2} \\
 & 15. \frac{1 + \frac{1}{x-1}}{1 + \frac{1}{x^2-1}} \\
 &= \frac{\frac{x-1+1}{x-1}}{\frac{x^2-1+1}{x^2-1}} = \frac{x(x+1)}{x^2-1} = \frac{x+1}{x} \\
 & 16. \frac{a - \frac{ab}{a+b}}{a + \frac{ab}{a-b}} = \frac{\frac{a^2 + ab - ab}{a+b}}{\frac{a^2 - ab + ab}{a-b}} \\
 &= \frac{a^2(a-b)}{a^2(a+b)} = \frac{a-b}{a+b} \\
 & 17. \frac{x-1 - \frac{5}{x+3}}{x+5 - \frac{35}{x+3}} \\
 &= \frac{\frac{x^2 + 2x - 8}{x+3}}{\frac{x^2 + 8x - 20}{x+3}} \\
 &= \frac{(x+4)(x-2)}{(x+10)(x-2)} = \frac{x+4}{x+10} \\
 & 18. \frac{a-4 + \frac{5a-11}{a+1}}{a^2 + 5a + 6 - \frac{7a-9}{a+3}} \\
 &= \frac{\frac{a^2 - 3a - 4 + 5a - 11}{a+1}}{\frac{a^2 + 5a + 6 - 7a + 9}{a+3}} \\
 &= \frac{(a+1)(a^2 - 2a - 3)}{(a+3)(a^2 + 2a - 15)} \\
 &= \frac{(a+1)(a-3)(a+1)}{(a+3)(a+5)(a-3)} \\
 &= \frac{(a+1)^2}{(a+3)(a+5)} = \frac{a^2 + 2a + 1}{a^2 + 8a + 15}
 \end{aligned}$$

EJERCICIO 18

$$\begin{aligned}
 & 1. \frac{1 + \frac{x+1}{x-1}}{\frac{1}{x-1} - \frac{1}{x+1}} \\
 &= \frac{\frac{x-1+x+1}{x-1}}{\frac{x+1-(x-1)}{x^2-1}} = \frac{2x(x+1)}{2} = x^2 + x \\
 & 2. \frac{\frac{1}{x-2} + \frac{2}{2x+6}}{\frac{x}{x} + \frac{x+1}{x+1}} \\
 &= \frac{\frac{x+1+2(x-1)}{x^2-1}}{\frac{x^2-1}{x(x+1)}} \\
 &= \frac{x(3x-1)}{(x-1)(3x^2+5x-2)} \\
 &= \frac{x(3x-1)}{(x-1)(3x-1)(x+2)} = \frac{x}{x^2+x-2} \\
 & 3. \frac{\frac{a}{a-b} - \frac{b}{a+b}}{\frac{a}{a-b} + \frac{b}{a+b}} \\
 &= \frac{\frac{a(a+b) - b(a-b)}{(a-b)(a+b)}}{\frac{b(a+b) + a(a-b)}{b(a-b)}} \\
 &= \frac{a^2 + ab - ab + b^2}{a+b} \cdot \frac{a^2 + b^2}{ab + b^2 + a^2 - ab} = \frac{a^2 + b^2}{a+b} \cdot \frac{a^2 + b^2}{a^2 + b^2} = \frac{b}{a+b} \\
 & 4. \frac{\frac{x+3}{x+4} - \frac{x+1}{x+2}}{\frac{x-1}{x+2} - \frac{x-3}{x+4}} \\
 &= \frac{\frac{(x+3)(x+2) - (x+1)(x+4)}{(x+4)(x+2)}}{\frac{(x-1)(x+4) - (x-3)(x+2)}{(x+2)(x+4)}} \\
 &= \frac{x^2 + 5x + 6 - x^2 - 5x - 4}{x^2 + 3x - 4 - x^2 + x + 6} \\
 &= \frac{2}{4x+2} = \frac{2}{2(2x+1)} = \frac{1}{2x+1}
 \end{aligned}$$

$$\begin{aligned}
 5. \quad & \frac{\frac{m^2}{n} + \frac{m^2 - n^2}{m+n}}{\frac{m-n}{n} + \frac{n}{m}} \\
 &= \frac{m^2(m+n) - n(m^2 - n^2)}{n(m+n)} \\
 &= \frac{m(m-n) + n^2}{nm} \\
 &= \frac{m^3 + m^2n - m^2n + n^3}{m+n} \\
 &= \frac{m^3 + n^3}{m+n} \\
 &= \frac{m^2 - mn + n^2}{m} \\
 &= \frac{(m+n)(m^2 - mn + n^2)}{m+n} \\
 &= \frac{m^2 - mn + n^2}{m} \\
 &= m^2 - mn + n^2 \cdot \frac{m}{m^2 - mn + n^2} = m
 \end{aligned}$$

$$\begin{aligned}
 6. \quad & \frac{\frac{a^2}{b^3} + \frac{1}{a}}{\frac{a}{b} - \frac{b-a}{b}} \\
 &= \frac{\frac{a^3 + b^3}{ab^3}}{a(a-b) - b(b-a)} \\
 &= \frac{(a+b)(a^2 - ab + b^2)}{b(a-b)} \\
 &= \frac{ab^3}{a^2 - ab - b^2 + ab} \\
 &= \frac{(a+b)(a^2 - ab + b^2)}{b(a-b)} \\
 &= \frac{ab^2}{a^2 - b^2} \\
 &= \frac{(a+b)(a^2 - ab + b^2)}{ab^2} \cdot \frac{a-b}{(a+b)(a-b)} \\
 &= \frac{a^2 - ab + b^2}{ab^2}
 \end{aligned}$$

$$\begin{aligned}
 7. \quad & \frac{1 + \frac{2x}{1+x^2}}{2x + \frac{2x^5 + 2}{1-x^4}} \\
 &= \frac{1+x^2+2x}{1+x^2} \\
 &= \frac{2x(1-x^4) + 2x^5 + 2}{(1+x^2)(1-x^2)} \\
 &= \frac{x^2 + 2x + 1}{2x - 2x^5 + 2x^5 + 2} \\
 &= \frac{(x+1)^2}{2(x+1)} \\
 &= \frac{(x+1)(1-x)}{2(x+1)} \\
 &= \frac{(x+1)^2(1-x)}{2} \\
 &= \frac{(x^2 + 2x + 1)(1-x)}{2} \\
 &= \frac{x^2 - x^3 + 2x - 2x^2 + 1 - x}{2} \\
 &= \frac{1 + x - x^2 - x^3}{2}
 \end{aligned}$$

$$\begin{aligned}
 8. \quad & \frac{\frac{x+y}{x+y} - \frac{x-y}{x+y}}{\frac{x+y}{x+y} - \frac{x+2y}{x+y}} \\
 &= \frac{(x+y)^2 - (x-y)^2}{(x+y)^2 - x(x+2y)} \\
 &= \frac{x^2 + 2xy + y^2 - x^2 - 2xy - y^2}{x^2 + 2xy + y^2 - x^2 - 2xy} \\
 &= \frac{4xy}{x} \\
 &= \frac{x-y}{y^2} \\
 &= \frac{4xy}{x-y} \cdot \frac{x}{y^2} = \frac{4x^2}{y(x-y)} = \frac{4x^2}{xy - y^2}
 \end{aligned}$$

$$\begin{aligned}
 9. \quad & \frac{\frac{a+x}{a-x} - \frac{b+x}{b-x}}{\frac{a-x}{a-x} - \frac{b-x}{b-x}} \\
 &= \frac{(a+x)(b-x) - (b+x)(a-x)}{(a-x)(b-x)} \\
 &= \frac{2(b-x) - 2(a-x)}{(a-x)(b-x)} \\
 &= \frac{ab - ax + bx - x^2 - ab + bx - ax + x^2}{2b - 2x - 2a + 2x} \\
 &= \frac{2bx - 2ax}{2b - 2a} = \frac{2x(b-a)}{2(b-a)} = x \\
 10. \quad & \frac{\frac{a}{a+x} - \frac{a}{2a+2x}}{\frac{a}{a-x} + \frac{a}{a+x}} = \frac{\frac{2a-a}{2(a+x)}}{\frac{a(a+x) + a(a-x)}{(a-x)(a+x)}} \\
 &= \frac{\frac{a}{2}}{\frac{a^2 + ax + a^2 - ax}{a-x}} \\
 &= \frac{\frac{a}{2}}{2a^2} = \frac{a}{2} \cdot \frac{a-x}{2a^2} = \frac{a-x}{4a}
 \end{aligned}$$

$$\begin{aligned}
 11. \quad & \frac{\frac{a+2b}{a-b} + \frac{b}{a}}{\frac{a+b}{a} + \frac{3b}{a-b}} = \frac{\frac{a(a+2b) + b(a-b)}{a(a-b)}}{\frac{(a+b)(a-b) + 3ab}{a(a-b)}} \\
 &= \frac{a^2 + 2ab + ab - b^2}{a^2 - b^2 + 3ab} \\
 &= \frac{a^2 + 3ab - b^2}{a^2 + 3ab - b^2} = 1
 \end{aligned}$$

$$\begin{aligned}
 12. \quad & 1 - \frac{7}{x} + \frac{12}{x^2} \\
 &= \frac{x^2 - 7x + 12}{x^2 - 16} \\
 &= \frac{(x-4)(x-3)}{(x-4)(x+4)} \\
 &= \frac{x-3}{x(x+4)} = \frac{x-3}{x^2 + 4x}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{\frac{a^2}{b} - \frac{b^2}{a}}{\frac{1}{b} + \frac{1}{a} + \frac{b}{a^2}} \\
 13. & \frac{\frac{a^3 - b^3}{ab}}{a^2 + ab + b^2} \\
 & = \frac{a^2 b}{a^2 + ab + b^2} \\
 & = \frac{a^3 - b^3}{a^2 + ab + b^2} \\
 & = \frac{a^3 - b^3}{a} \\
 & = (a-b)(a^2 + ab + b^2) \cdot \frac{a}{a^2 + ab + b^2} \\
 & = a(a-b) = a^2 - ab
 \end{aligned}$$

$$\begin{aligned}
 & \frac{x-2y - \frac{4y^2}{x+y}}{x-3y - \frac{5y^2}{x+y}} \\
 14. & \frac{(x-2y)(x+y) - 4y^2}{(x-3y)(x+y) - 5y^2} \\
 & = \frac{x^2 - xy - 6y^2}{x^2 - 2xy - 8y^2} \\
 & = \frac{(x-3y)(x+2y)}{(x-4y)(x+2y)} = \frac{x-3y}{x-4y}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{\frac{2}{1-a} + \frac{2}{1+a}}{\frac{1}{1+a} - \frac{1}{1-a}} \\
 15. & \frac{2(1+a) + 2(1-a)}{(1-a)(1+a)} \\
 & = \frac{2(1-a) - 2(1+a)}{(1+a)(1-a)} \\
 & = \frac{2+2a+2-2a}{2-2a-2-2a} = -\frac{4}{4a} = -\frac{1}{a}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{\frac{1}{x+y+z} - \frac{1}{x-y+z}}{\frac{1}{x-y+z} - \frac{1}{x+y+z}} \\
 16. & \frac{\frac{x-y+z-x-y-z}{(x+y+z)(x-y+z)}}{\frac{x-y+z-x-y-z}{(x-y+z)(x+y+z)}} = \frac{-2y}{2y} = -1
 \end{aligned}$$

Prohibida su venta
Material de Apoyo exclusivo
para uso didáctico

$$\begin{aligned}
 17. \quad & \frac{1 + \frac{2b+c}{a-b-c}}{1 - \frac{c-2b}{a-b+c}} \\
 &= \frac{\frac{a-b-c+2b+c}{a-b-c}}{\frac{a-b+c-c+2b}{a-b+c}} \\
 &= \frac{\frac{a+b}{a-b-c}}{\frac{a+b}{a-b+c}} \\
 &= \frac{a+b}{a-b-c} \cdot \frac{a-b+c}{a+b} = \frac{a-b+c}{a-b-c}
 \end{aligned}$$

$$\begin{aligned}
 18. \quad & \frac{\frac{a}{1-a} + \frac{1-a}{a}}{1-a - \frac{a}{1-a}} \\
 &= \frac{\frac{a^2 + (1-a)^2}{a(1-a)}}{\frac{(1-a)^2 - a^2}{a(1-a)}} \\
 &= \frac{a^2 + 1 - 2a + a^2}{1 - 2a + a^2 - a^2} = \frac{2a^2 - 2a + 1}{1 - 2a}
 \end{aligned}$$

$$\begin{aligned}
 19. \quad & \frac{x+1 - \frac{6x+12}{x+2}}{x-4 + \frac{11x-22}{x-2}} \\
 &= \frac{x+1 - \frac{6(x+2)}{x+2}}{x-4 + \frac{11(x-2)}{x-2}} = \frac{x-5}{x+7} = 1
 \end{aligned}$$

$$20. \quad 1 + \frac{1}{x} = \frac{1}{\frac{1}{x+1}} = 1 \cdot \frac{x}{x+1} = \frac{x}{x+1}$$

$$\begin{aligned}
 21. \quad & \frac{1}{1 + \frac{1}{1 - \frac{1}{x}}} = \frac{1}{1 + \frac{1}{\frac{x-1}{x}}} \\
 &= \frac{1}{1 + \frac{x}{x-1}} \\
 &= \frac{1}{\frac{x-1+x}{x-1}} \\
 &= \frac{1}{\frac{2x-1}{x-1}} = 1 \cdot \frac{x-1}{2x-1} = \frac{x-1}{2x-1}
 \end{aligned}$$

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$$\begin{aligned}
 22. \quad & 1 - \frac{1}{2 + \frac{1}{\frac{x-1}{3}}} \\
 &= 1 - \frac{1}{2 + \frac{1}{\frac{x-3}{3}}} \\
 &= 1 - \frac{1}{2 + \frac{3}{x-3}} \\
 &= 1 - \frac{1}{\frac{2x-3}{x-3}} \\
 &= 1 - \frac{x-3}{2x-3} \\
 &= \frac{2x-3-x+3}{2x-3} = \frac{x}{2x-3}
 \end{aligned}$$

$$\begin{aligned}
 23. \quad & \frac{2}{1 + \frac{2}{1 + \frac{2}{x}}} \\
 &= \frac{2}{1 + \frac{2}{\frac{x+2}{x}}} \\
 &= \frac{2}{1 + \frac{2x}{x+2}} \\
 &= \frac{2}{\frac{x+2+2x}{x+2}} \\
 &= \frac{2}{3x+2} = 2 \cdot \frac{x+2}{3x+2} = \frac{2x+4}{3x+2}
 \end{aligned}$$

$$\begin{aligned}
 24. \quad & \frac{1}{x - \frac{x}{x - \frac{x^2}{x+1}}} \\
 &= \frac{1}{x - \frac{x}{\frac{x^2+x-x^2}{x+1}}} \\
 &= \frac{1}{x - \frac{x}{\frac{x^2+x}{x}}} \\
 &= \frac{1}{\frac{x^2-x^2-x}{x}} = \frac{1}{\frac{-x}{x}} = -1
 \end{aligned}$$

$$\begin{aligned}
 25. \quad & \frac{1}{a+2 - \frac{a+1}{a - \frac{1}{a}}} \\
 &= \frac{1}{a+2 - \frac{a+1}{\frac{a^2-1}{a}}} = \frac{1}{a+2 - \frac{a(a+1)}{(a+1)(a-1)}} = \frac{1}{a+2 - \frac{a}{a-1}} = \frac{1}{\frac{(a+2)(a-1)-a}{a-1}} = \frac{1}{\frac{a^2+a-2-a}{a-1}} = 1 \cdot \frac{a-1}{a^2-2} = \frac{a-1}{a^2-2}
 \end{aligned}$$

$$\begin{aligned}
 26. \quad & \frac{x-1}{x+2 - \frac{x^2+2}{x - \frac{x-2}{x+1}}} = \frac{x-1}{x+2 - \frac{x^2+2}{\frac{x^2+x-x+2}{x+1}}} = \frac{x-1}{x+2 - \frac{x^2+2}{\frac{x^2+2}{x+1}}} = \frac{x-1}{x+2 - \frac{(x^2+2)(x+1)}{x^2+2}} = \frac{x-1}{x+2-x-1} = x-1
 \end{aligned}$$

Prohibida su venta
Material de Apoyo exclusivo
para uso didáctico

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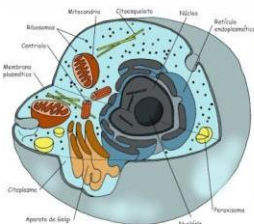
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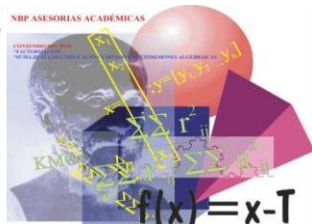


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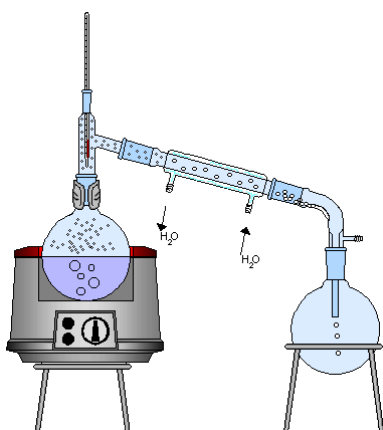
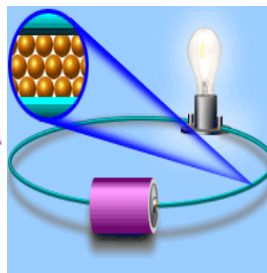
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